Chapter 25
Optical Instruments
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Basic parts of a camera:

- Lens
- Light-tight box
- Shutter
- Film or electronic sensor
25-1 Cameras, Film, and Digital

A digital camera uses CCD (charge-coupled device) or CMOS (complementary metal oxide semiconductor) sensors instead of film. The digitized image is sent to a processor for storage and later retrieval.

When exposed to light, pixels accumulate charge. This charge is measured.

In the most common arrays, Bayer mosaic, color is achieved by red, blue and green filters over alternating pixels.
Camera adjustments:

- **Shutter speed**: controls the amount of time light enters the camera. A faster shutter speed makes a sharper picture.
- **f-stop**: controls the maximum opening of the shutter. This allows the right amount of light to enter to properly expose the film, and must be adjusted for external light conditions.
- **Focusing**: this adjusts the position of the lens so that the image is positioned on the film.

\[ f\text{-stop} = \frac{f}{D} \]

(f is the focal length and D is the lens opening)
There is a certain range of distances over which objects will be in focus; this is called the depth of field of the lens. Objects closer or farther will be blurred. Objects at the far distance always have an image at the focal point.

Rays from nearby object (in focus)

Rays from distant object

“Circle of confusion” for distant object (greatly exaggerated)
Example

• Haw far must a 50.0 mm-focal-length camera lens be moved from its infinity setting to sharply focus an object 3 m away?

• Answer: 0.8 mm away from the film or digital sensor.

• Solution:

Use thin lens equation:

\[
\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \quad \Rightarrow \quad d_i = 50.8 \, mm
\]

\[
\Delta = d_i - f = 0.8 \, mm
\]
The human eye resembles a camera in its basic functioning, with an adjustable lens, the iris, and the retina.

Vitreous humor – gel-like substance with index of refraction n=1.337;

Diaphragm – iris (the colored part of the eye);

Pupil – hole in the iris for light to pass in the eye

Retina – analog of sensor or film
Most of the refraction is done at the surface of the cornea \((n=1.376)\) at its interface with air \((n=1)\); the lens of the eye \((n=1.386 \text{ to } 1.406)\) makes small adjustments to focus at different distances.

This is accomplished by the ciliary muscles, which change the curvature of the lens so its focal length is changed.
Near point: closest distance at which eye can focus clearly. Normal is about 25 cm.

Far point: farthest distance at which object can be seen clearly. Normal is at infinity.

Nearsightedness: far point is too close.

Farsightedness: near point is too far away.
Nearsightedness can be corrected with a diverging lens. Optometrists specify the power $P=1/f$ given in diopters.
And farsightedness with a converging lens. Optometrists specify the power $P = \frac{1}{f}$ given in diopters.
Vision is blurry underwater because light rays are bent much less than they would be if entering the eye from air. This can be avoided by wearing goggles.
Example

- A person’s right eye can see objects clearly only if they are between 25 cm and 85 cm away. (a) What power of contact lens is required so that objects far away are sharp? (b) what will be the near point with the lens in place?

(a) The lens should put the image of an object at infinity at the person’s far point of 85 cm. Note the image is still in front of the eye, so the image distance is negative. Use Eqs. 23–7 and 23–8.

\[ P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \left( -\frac{1}{0.85m} \right) = -1.2 \text{ D} \]

(b) To find the near point with the lens in place, we find the object distance to form an image 25 cm in front of the eye.

\[ P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow d_o = \frac{d_i}{Pd_i - 1} = \frac{-0.25 \text{ m}}{-1.2D \cdot (-0.25 \text{ m}) - 1} = 0.35 \text{ m} \]
25-3 Magnifying Glass

A magnifying glass (simple magnifier) is a converging lens. It allows us to focus on objects closer than the near point, so that they make a larger, and therefore clearer, image on the retina.

A comparison of these two cases reveals that the angle the object subtends at the eye is much larger when the magnifier is used.

$$M = \frac{\theta'}{\theta}$$

$$\theta = \frac{h}{N} \text{ and } \theta' = \frac{h}{d_o}.$$ So

$$M = \frac{N}{f} \text{ (if eye is focused at infinity; } N=25 \text{ cm for normal eye)}$$
25-3 Magnifying Glass

The power of a magnifying glass is described by its angular magnification:

\[ M = \frac{\theta'}{\theta} \quad (25-1) \]

If the eye is relaxed (\( N \) is the near point distance and \( f \) the focal length):

\[ M = \frac{\theta'}{\theta} = \frac{h/f}{h/N} = \frac{N}{f}. \quad (25-2a) \]

If the eye is focused at the near point:

\[ M = \frac{N}{f} + 1. \quad (25-2b) \]

The shorter is the focus length, the stronger is magnification.
25-4 Telescopes

A refracting telescope consists of two lenses at opposite ends of a long tube. The objective lens is closest to the object, and the eyepiece is closest to the eye. The objective lens forms the image, and the eyepiece greatly magnifies the image.

The magnification is given by:

\[ M = \frac{\theta'}{\theta} = \frac{(h/f_e)}{(h/f_o)} = -\frac{f_o}{f_e} \]  (25-3)

where \( f_o \) is the focal length of the objective lens, and \( f_e \) is the focal length of the eyepiece.
Parallel light from one point on a distant object is brought to a focus by the objective lens in its focal plane. This image $I_1$ is magnified by the eyepiece to form the final image $I_2$.

\[
M = \frac{\theta'}{\theta} = \frac{h/f_e}{h/f_o} = -\frac{f_o}{f_e}
\]
Astronomical telescopes need to gather as much light as possible, meaning that the objective must be as large as possible. Hence, mirrors are used instead of lenses, as they can be made much larger and with more precision.
A terrestrial telescope, used for viewing objects on Earth, should produce an upright image. Here are two models, a Galilean type (top) and a spyglass (bottom).

In the second type all lenses are convex, three lenses are used to make the image upright.
25-5 Compound Microscope

A compound microscope also has an objective and an eyepiece; it is different from a telescope in that the object is placed very close to the eyepiece. The image $I_1$ formed by objective lens is real and enlarged. The eyepiece is positioned in such a way, that this image is near the eyepiece focal point $F_e$, so it can be significantly magnified by the eyepiece into a very large virtual image $I_2$.
25-5 Compound Microscope

The image I1 formed by the objective lens is a factor $m_0$ larger than the object itself: 

$$m_0 = \frac{h_i}{h_o} = \frac{d_i}{d_o} = \frac{(l-f_e)}{d_o}$$

If we assume that the eye is relaxed, the eyepiece angular magnification $M_e$ is $M_e = N/f_e$.

Final magnification $M$ is the product of $m$ and $M_e$.

The magnification is given by:

$$M = M_e m_o = \left( \frac{N}{f_e} \right) \left( \frac{l - f_e}{d_o} \right)$$  \hspace{1cm} (25-6a)

$$\approx \frac{N\ell}{f_e f_o}.$$ \hspace{1cm} [f_o and f_e \ll \ell]  \hspace{1cm} (25-6b)
Example

The eyepiece of a compound microscope has a focal length of 2.8 cm and objective lens has \( f = 0.74 \) cm. If an object is placed 0.79 cm from objective lens, calculate (a) distance between between the lenses when the microscope is adjusted for a relaxed eye, and (b) the total magnification.

• (a) We find the image distance from the objective using Eq. 23–8. For the final image to be at infinity (viewed with a relaxed eye), the objective image distance must be at the focal distance of the eyepiece. We calculate the distance between the lenses as the sum of the objective image distance and the eyepiece focal length.

\[
\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_o} \quad \Rightarrow \quad d_{i1} = \frac{f_o d_{o1}}{d_{o1} - f_o} = \frac{(0.740 \text{ cm})(0.790 \text{ cm})}{0.790 \text{ cm} - 0.740 \text{ cm}} = 11.69 \text{ cm}
\]

\[
\ell = d_{i1} + f_e = 11.69 \text{ cm} + 2.80 \text{ cm} = 14.49 \text{ cm} \approx [14 \text{ cm}]
\]

• (b) We use Eq. 25–6a to calculate the total magnification.

\[
M = \frac{N}{f_e} \left( \frac{\ell - f_e}{d_o} \right) = \frac{(25 \text{ cm})(14.49 \text{ cm} - 2.80 \text{ cm})}{(2.80 \text{ cm})(0.790 \text{ cm})} = 132 \times \approx [130 \times]
\]

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25-6 Aberrations of Lenses and Mirrors

Spherical aberration: rays far from the lens axis do not focus at the focal point.

Solutions: compound-lens systems; use only central part of lens
25-6 Aberrations of Lenses and Mirrors

Distortion: caused by variation in magnification with distance from the lens. Barrel and pincushion distortion:

They occur for monochromatic light and are referred to as chromatic aberrations.
Chromatic aberration: light of different wavelengths has different indices of refraction and focuses at different points.
25-6 Aberrations of Lenses and Mirrors

Solution: Achromatic doublet, made of lenses of two different materials
Resolution is the distance at which a lens can barely distinguish two separate objects.

Resolution is limited by aberrations and by diffraction. Aberrations can be minimized, but diffraction is unavoidable; it is due to the size of the lens compared to the wavelength of the light.
25-7 Limits of Resolution; Circular Apertures

For a circular aperture of diameter $D$, the central maximum has an angular width:

$$\theta = \frac{1.22\lambda}{D}$$

Since the resolution is directly proportional to the wavelength and inversely proportional to the diameter, radio telescopes are built to be very large.
The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.
25-8 Resolution of Telescopes and Microscopes; the $\lambda$ Limit

For microscopes, assuming the object is at the focal point, the resolving power is given by:

$$\text{RP} = s = f\theta = \frac{1.22\lambda f}{D} \quad (25-8)$$

Typically, the focal length of a microscope lens is half its diameter, which shows that it is not possible to resolve details smaller than the wavelength being used.

$$\text{RP} \approx \frac{\lambda}{2} \quad (25-9)$$
25-9 Resolution of the Human Eye and Useful Magnification

The human eye can resolve objects that are about 1 cm apart at a distance of 20 m, or 0.1 mm apart at the near point.

This limits the useful magnification of a light microscope to about 500×–1000×.
Example

Suppose that you have to construct a telescope that can resolve features 6.5 km across the Moon, 384000 km away. You have a 2m-focal-length objective lens whose diameter is 11 cm. What focal-length eyepiece is needed if your eye can resolve objects 0.1 mm apart at a distance of 25 cm? What is the resolution limit set by the size of the objective lens? Use $\lambda = 560$ nm.

To find the focal length of the eyepiece we use Eq. 25–3, where the objective focal length is 2.00 m, $\theta'$ is the ratio of the minimum resolved distance and 25 cm, and $\theta$ is the ratio of the object on the Moon and the distance to the Moon.

$$\frac{f_o}{f_e} = \frac{\theta'}{\theta} \rightarrow f_e = f_o \frac{\theta}{\theta'} = f_o \frac{(d_o/\ell)}{(d/N)} = (2.0 \text{ m}) \frac{(6.5 \text{ km}/384,000 \text{ km})}{(0.10 \text{ mm}/250 \text{ mm})} = 0.0846 \text{ m} \approx 8.5 \text{ cm}$$

We use Eq. 25–7 to determine the resolution limit.

$$\theta = 1.22 \frac{\lambda}{D} = 1.22 \frac{560 \times 10^{-9} \text{ m}}{0.11 \text{ m}} = 6.2 \times 10^{-6} \text{ rad}$$
In addition to sufficient resolving power, a microscope must be able to distinguish the object from its background. The difference in brightness between the image of an object and the image of surroundings is called contrast.
One way to do this is by using an interference microscope, which can detect objects by the change in wavelength as the light passes through them. Light from a source is split into 2 equal beams by half-silvered mirror MS1. One beam passes through the object, and the second (comparison beam) passes through an identical system without the object. The two meet again and are superimposed by the half-silvered mirror MS2 before entering the eyepiece. The path length can be adjusted in such a way that the background will look dark (destructive interference), while for an object there will be some small phase shift and the interference will not be completely destructive.
25-11 X-Rays and X-Ray Diffraction

**X-ray crystallography** is a tool used for identifying the atomic and molecular structure of a crystal, in which the crystalline atoms cause a beam of incident X-rays to diffract into many specific directions. By measuring the angles and intensities of these diffracted beams, a crystallographer can produce a three-dimensional picture of the density of electrons within the crystal. From this electron density, the mean positions of the atoms in the crystal can be determined, as well as their chemical bonds, their disorder and various other information.
25-11 X-Rays and X-Ray Diffraction

X-ray crystallography has been fundamental in the development of many scientific fields. In its first decades of use, this method determined

1) the size of atoms;
2) the lengths and types of chemical bonds;
3) the atomic-scale differences among various materials, especially minerals and alloys.

The method also revealed the structure and function of many biological molecules, including vitamins, drugs, proteins and nucleic acids such as DNA.
Crystals are regular arrays of atoms, and X-rays can be considered waves of electromagnetic radiation. Atoms scatter X-ray waves, primarily through the atoms' electrons. Just as an ocean wave striking a lighthouse produces secondary circular waves emanating from the lighthouse, so an X-ray striking an electron produces secondary spherical waves emanating from the electron. This phenomenon is known as elastic scattering. A regular array of electrons produces a regular array of spherical waves. Although these waves cancel one another out in most directions through destructive interference, they add constructively in a few specific directions:

$$2d \sin \phi = n\lambda$$

(Bragg’s equation)
Example

X-rays of wavelength 0.138 nm fall on crystal whose atoms, lying in planes, are spaces at 0.285 nm apart. At what angle $\phi$ must the X-ray be directed if the first diffraction maximum is to be observed?

\[
2d \sin \phi = n\lambda \\
\sin \phi = \frac{\lambda}{2d} = \frac{0.138 \text{ nm}}{2 \cdot 0.285 \text{ nm}} = 0.242 \\
\phi = 14^\circ
\]
25-12 X-Ray Imaging and Computed Tomography (CT Scan)

A conventional X-ray is essentially a shadow; there are no lenses involved.
X-ray computed tomography (x-ray CT) is a technology that uses computer-processed x-rays to produce tomographic images (virtual 'slices') of specific areas of the scanned object, allowing the user to see inside without cutting. Digital geometry processing is used to generate a three-dimensional image of the inside of an object from a large series of two-dimensional radiographic images taken around a single axis of rotation.

The CT scanner was invented in 1972 by the British engineer Godfrey N. Hounsfield (later Sir Godfrey) and the South African (later American) physicist Alan Cormack. CT scanning was already in general use by 1979, the year Hounsfield and Cormack were awarded the Nobel Prize in Medicine or Physiology for its development.
25-12 X-Ray Imaging and Computed Tomography (CT Scan)

Computed tomography uses a narrow beam of X-rays, and takes measurements at many different angles. The measurements are sent to a computer, which combines them into a detailed image.
A fan-beam CT scanner is similar to the one on the previous slide, but uses multiple beams and detectors to complete the scan more quickly.
25-12 X-Ray Imaging and Computed Tomography (CT Scan)

Images can be color-coded to enhance areas of increased X-ray absorption; this improves the image considerably. The size of the pixels is determined by the size of the detectors.
Summary of Chapter 25

• Camera lens forms image by letting light through a shutter; can be adjusted for different light levels using f-stop and focused by moving lens

• Human eye forms image by letting light through pupil; adjusts to different light levels using iris and focuses by changing thickness of lens

• Nearsighted vision is corrected by diverging lens, farsighted by converging
Summary of Chapter 25

- Simple magnifier: object at focal point
- Angular modification:
  \[ M = \frac{\theta'}{\theta} = \frac{h/f}{h/N} = \frac{N}{f}. \]
- Astronomical telescope: objective and eyepiece; object infinitely far away
- Magnification:
  \[ M = \frac{\theta'}{\theta} = \frac{(h/f_e)}{(h/f_o)} = -\frac{f_o}{f_e}. \]
Summary of Chapter 25

• Spherical aberration: rays far from axis do not go through focal point
• Chromatic aberration: different wavelengths have different focal points
• Resolution of optical devices is limited by diffraction