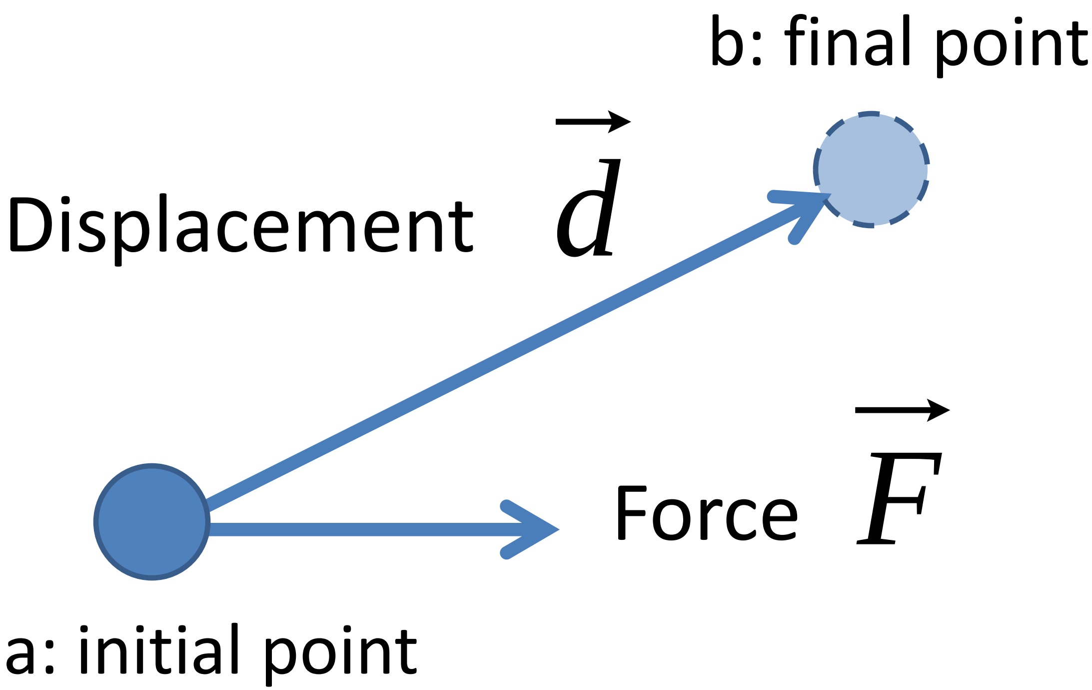
# Chapter 17

Electric Potential

# Force and Work: Reminder



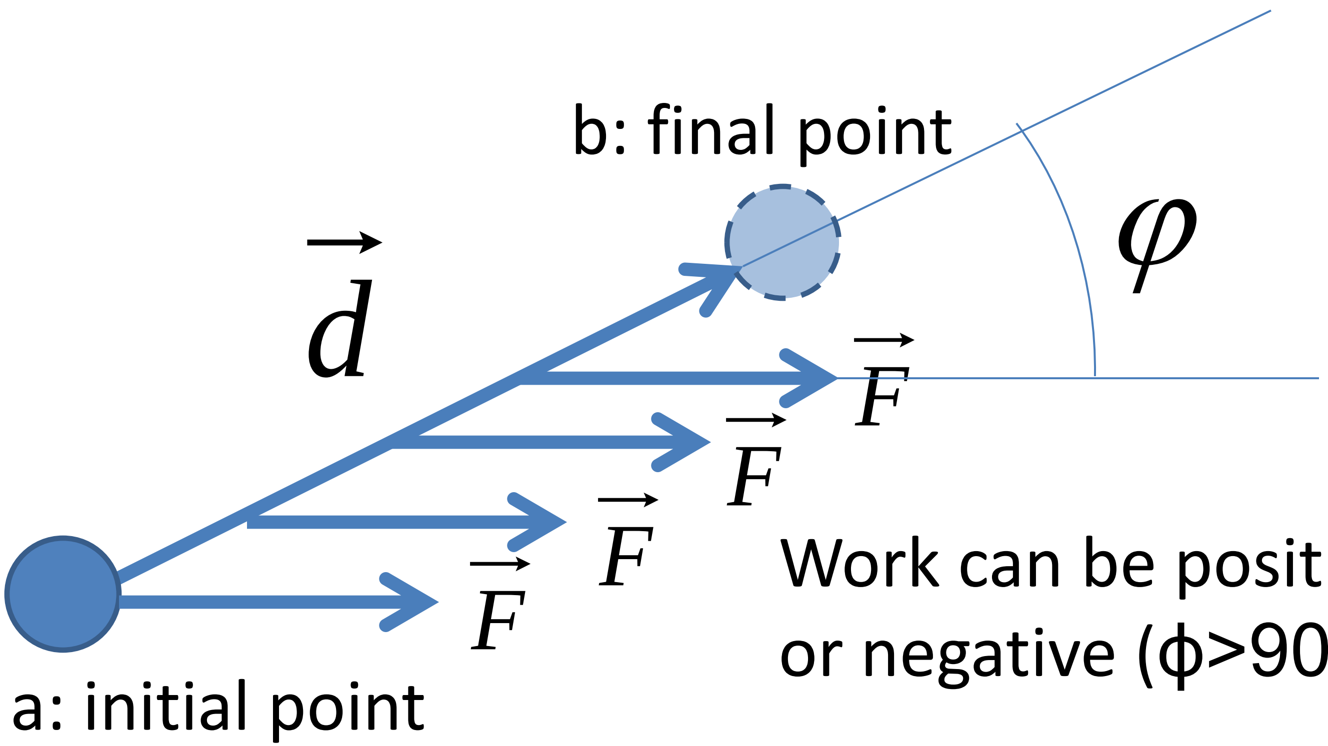

Reminder from Mechanics:

- if there is a force acting on an object (e.g. electric force), this force may do some work when the object moves

# Force and Work

$$W_{a \to b} = Fd\cos\varphi$$

that's why direction matters

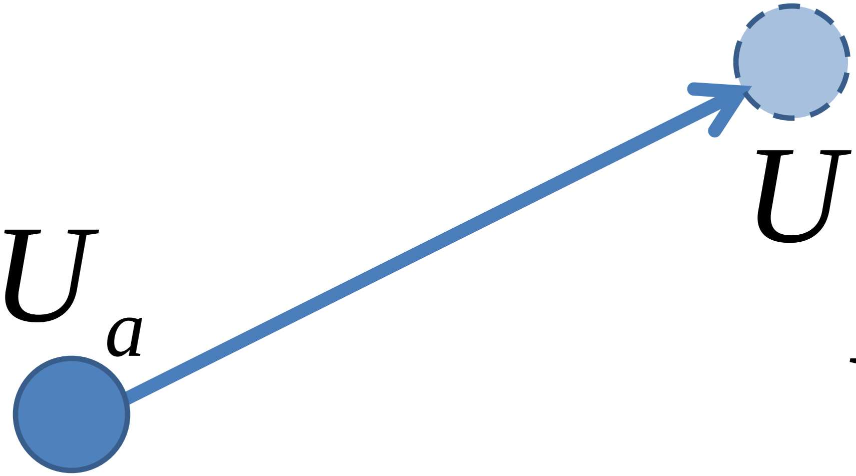



Work can be positive (ϕ<90º) or negative (ϕ>90º)

# Conservative Force

- If a force is conservative, there is something called potential energy U which changes from point to point

b: final point



$U_b$

$U_a$

a: initial point

Work does not depend on the path!

$$W_{a\to b} = U_a - U_b$$

# Work-Energy Theorem

- Change in kinetic energy = work

$$K_b - K_a = W_{a\to b}$$

$$K_b - K_a = U_a - U_b$$

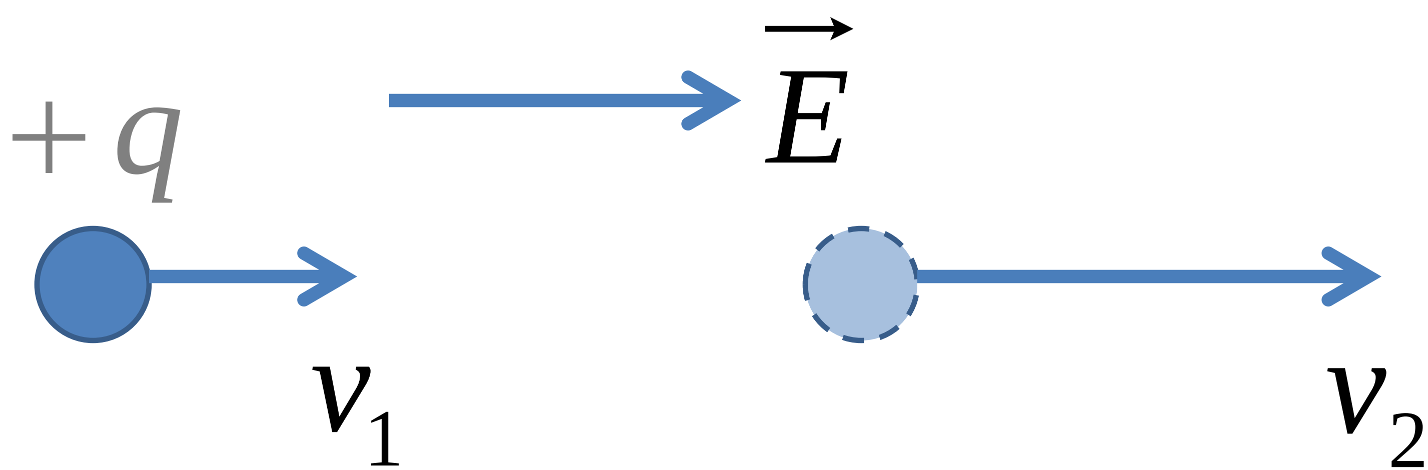
$$K_a + U_a = K_b + U_b$$

total energy doesn't change

# Electric Potential Energy

- If charge *q* is placed in electric field *E*, the force acting on it is $\vec{F} = q\vec{E}$

$$U_a - U_b = W_{a\to b} = qEd\cos\varphi$$



# Electric Potential

- Force → Field = Force divided by probe charge *q*
  - Field is independent of *q*
- Potential energy → Potential = Potential energy divided by probe charge *q*
  - Potential is independent of *q*

$$V = \frac{U}{q}$$

[*V*] = Volt 1 V = 1 J / C

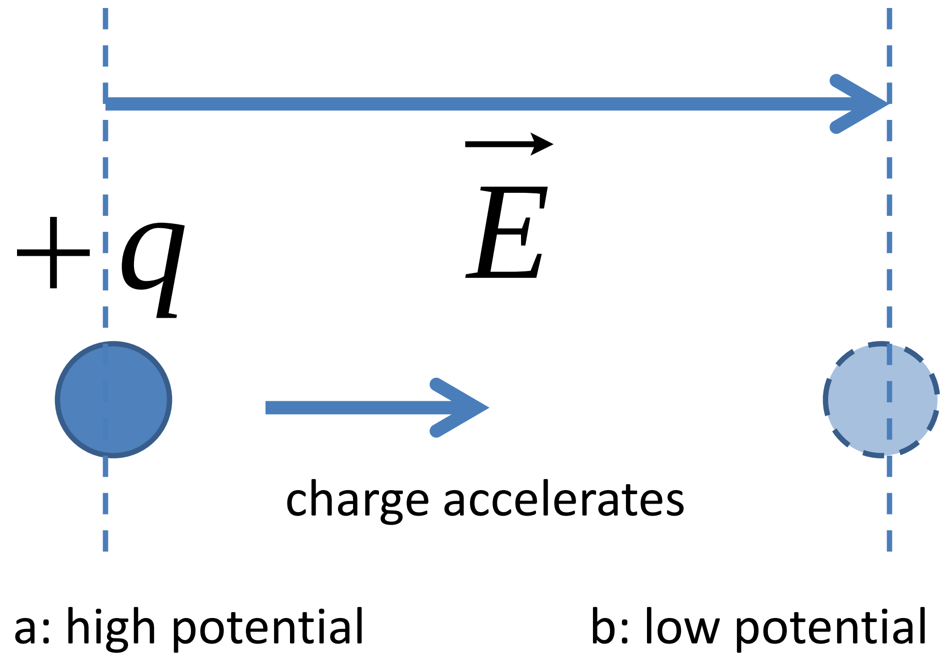
# Electric Potential Difference

- Like potential energy, potential itself is meaningless, what makes sense is the potential difference

$$V_{ba} = -\frac{W_{a\to b}}{q}$$

potential difference ("voltage") between points a and b

$$V_{ba} = V_b - V_a$$



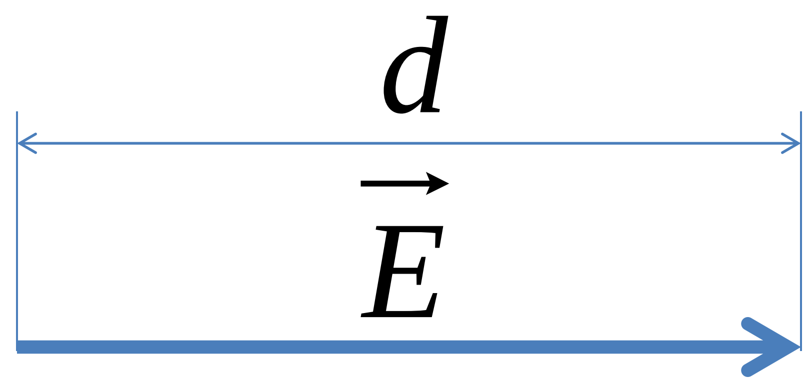
# Relation between Electric Potential Difference and Electric Field

$$W_{a \to b} = qEd\cancel{\cos\varphi}$$

consider a probe charge moving parallel to electric field lines

$$V_{ba} = -\frac{W_{a \to b}}{q}$$

$$\boxed{V_{ba} = -Ed}$$



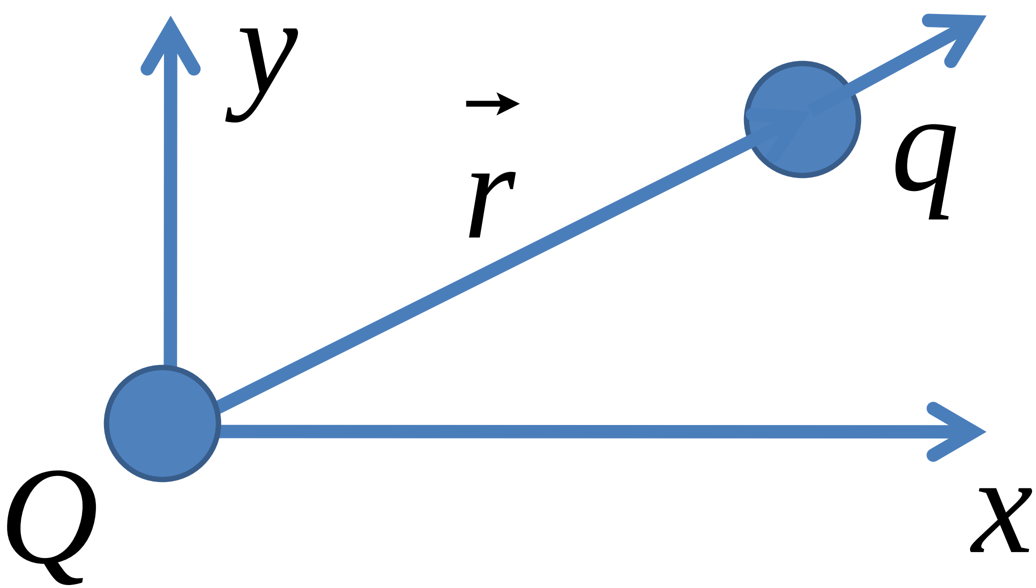

a: high potential

b: low potential

# Field of Point Charge

- Charge produces electric field which can act on another charge

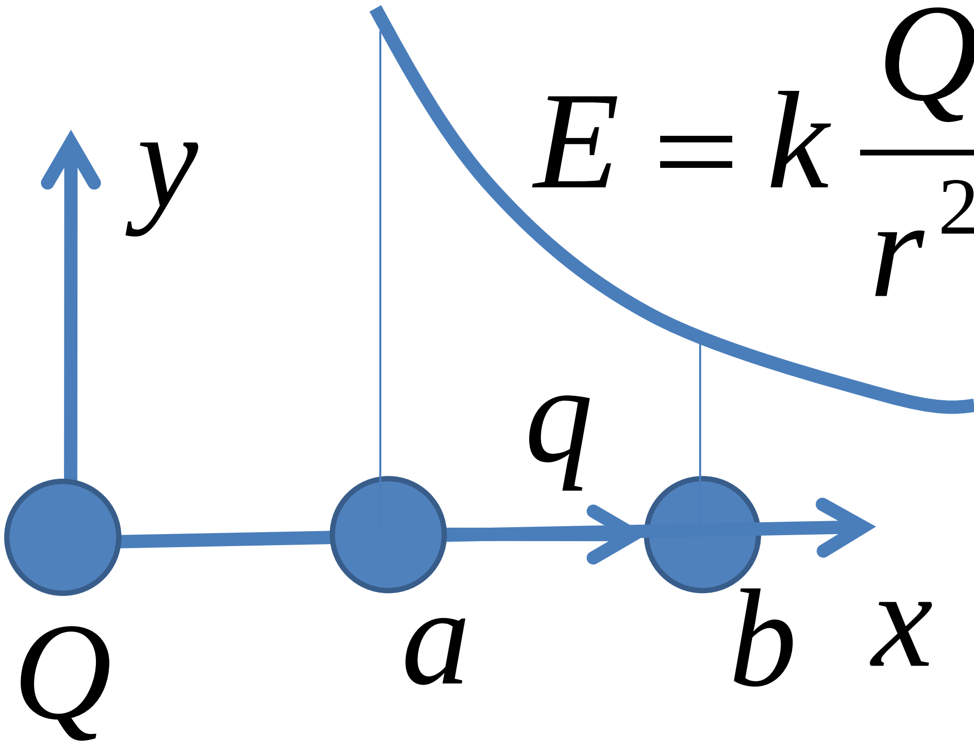
there is potential energy due to interaction of two charges




- Field is non-uniform
- Field is not constant

$$E = k\frac{Q}{r^2}$$

# Work in Field of Point Charge




$$E = k\frac{Q}{r^2}$$

ϕ=0, OK

$$W = qEd\cos\varphi$$

E varies with *x*, not OK !

Need calculus to compute result

# Work in Field of Point Charge

$$W_{a\to b} = kQq\left(\frac{1}{a} - \frac{1}{b}\right) = k\frac{Qq}{a} - k\frac{Qq}{b}$$

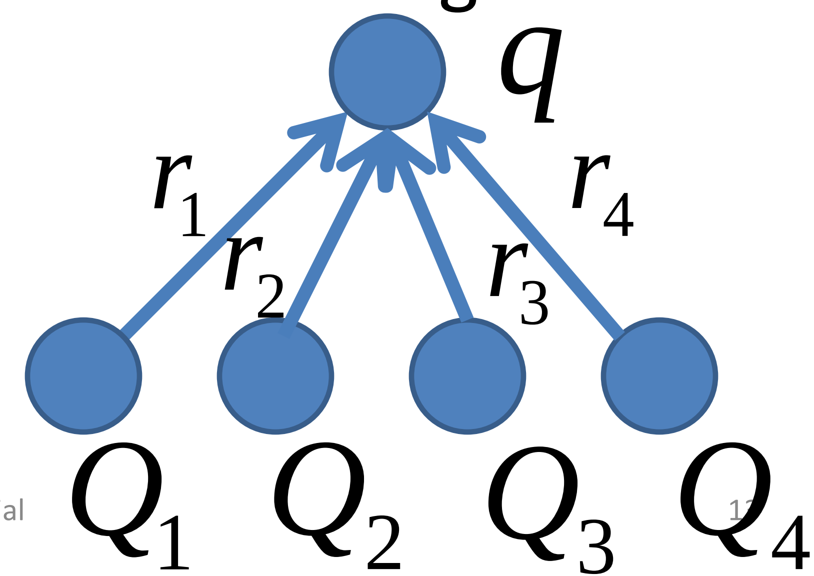
$$U = k\frac{Qq}{r}$$

Formula valid for any combination of q, q' signs

# More about potential energy

- U is always defined w.r.t. U at some point chosen by convention (only $U_a$-$U_b$ has physical meaning)
  - our choice: U=0 at r=∞
- What if there are more than one charge?

$$U = kq\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots\right)$$



# Potential of a Point Charge

$$V = \frac{U}{q} = k\frac{Q}{r}$$

- Both potential and electric field are independent from test charge
- Potential is a scalar, field is a vector

For many point charges:

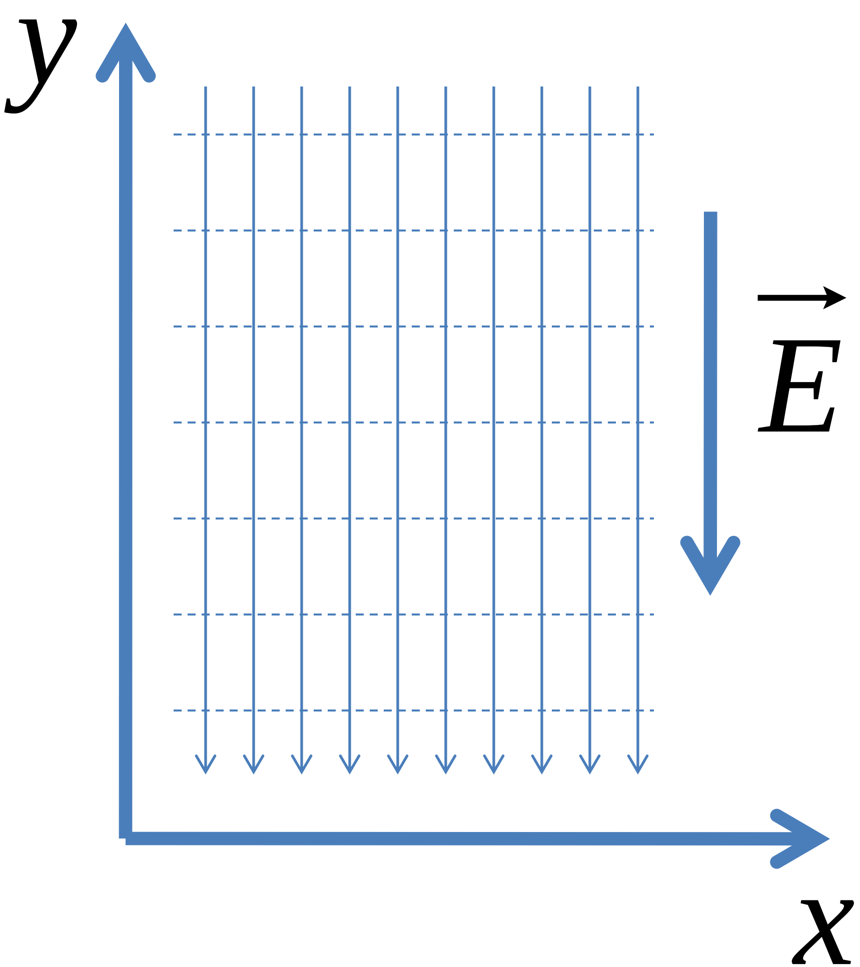
$$V = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots\right)$$

# Equipotential Surfaces

electric field line: imaginary line which in each point is tangent to electric field vector
equipotential surface: potential is the same at every point

- two equipotential surfaces never touch or intersect
- equipotential surfaces are mutually perpendicular to field lines

# Uniform Field




$$V = \frac{U}{q} = Ey$$

$$V = const \Rightarrow$$

$$y = const$$

equipotential surfaces = planes perpendicular to y axis

# Point Charge

$$V = k\frac{Q}{r}$$

$$V = const \Rightarrow$$

$$r = const$$

equipotential surfaces = spheres with centers at origin

# Dipole

If charges +q and −q are placed at y=+a and y=−a then plane y=0 is an equipotential surface

Proof:

$$V = k\left(\frac{Q}{r_1} - \frac{Q}{r_2}\right)$$

at y=0 $r_1=r_2$, so V=0

# Potential Gradient

- The magnitude of the electric field at any point on an equipotential surface = rate of change of potential

$$E = -\frac{\Delta V}{\Delta s}$$

as a point moves along electric field, potential decreases

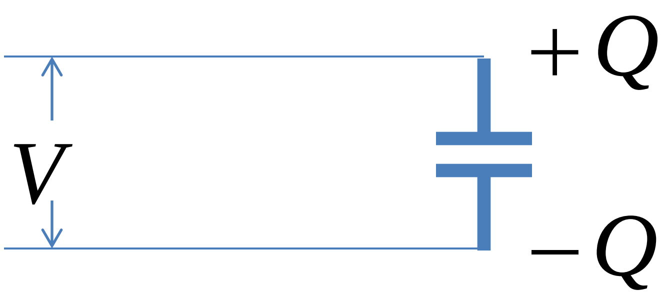
electric field is a vector, so what about direction?

$$\Delta s = (\Delta x, \Delta y, \Delta z)$$

$$\vec{E} = \left( -\frac{\Delta V}{\Delta x}, -\frac{\Delta V}{\Delta y}, -\frac{\Delta V}{\Delta z} \right)$$

# Capacitor and capacitance

- Capacitor = device which stores electric charge
  - Capacitor ≠ battery!
- Capacitor = two conductors separated by an insulator
  - generally, conductor 1 has charge $q_1$, and conductor 2 has charge $q_2$, but we'll always assume $q_1=Q=-q_2$, so the net charge is 0
- Capacitance C = capacitor's ability to store charge




$$C = \frac{Q}{V}$$

[C] = Farad, 1 F = 1 C / V

# Parallel-Plate Capacitor

$$\sigma = \frac{Q}{A}$$ surface charge density

$$E = \frac{\sigma}{\varepsilon_0}$$ this can be proved using Gauss' law

$$V = Ed$$

$$C = \frac{Q}{V}$$

$$C = \varepsilon_0 \frac{A}{d}$$

$\varepsilon_0 = 1/4\pi k = 8.854 \times 10^{-12}\ C^2/N \cdot m^2$

# Dielectrics

- If there is some material between the capacitor plates, the capacitance increases due to polarization

$$K = \frac{C}{C_0} \qquad V = \frac{V_0}{K} \qquad E = \frac{E_0}{K}$$

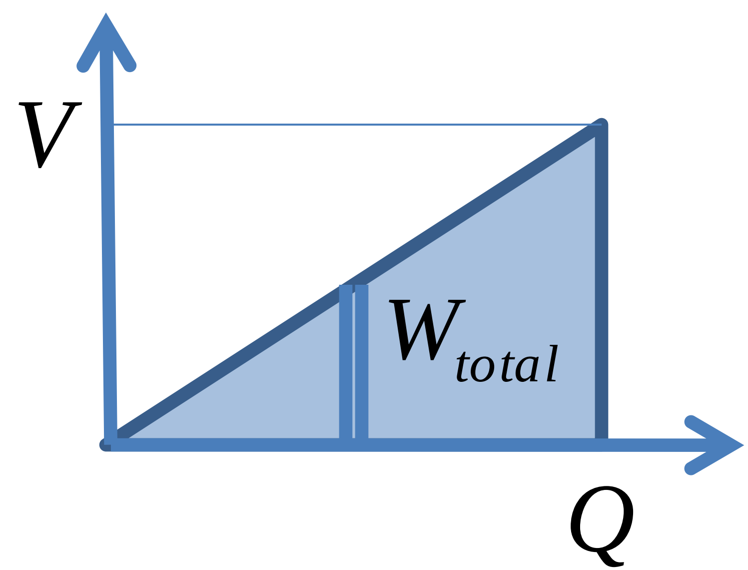
K = dielectric constant (a pure number depending on material)
If we insert material inside capacitor charged with charge Q, the voltage decreases by K

# Electric Field Energy

- Potential energy of a capacitor = work needed to charge it
  - If there is already charge Q and we want to add more charge ΔQ then we need to do work ΔW=VΔQ

$$\Delta W = V\Delta Q = \frac{Q\Delta Q}{C}$$

$$U = W_{total} = \frac{V}{2}Q = \frac{Q^2}{2C} = \frac{CV^2}{2}$$



# Electric Field Energy Density

- Energy density u = energy U per unit volume v

$$v = Ad$$

$$u = \frac{U}{v} = \frac{CV^2}{2Ad}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$V = Ed$$

$$u = \frac{\varepsilon_0 E^2}{2}$$

there is nothing related to a capacitor in this formula – it's also valid for a field in vacuum!