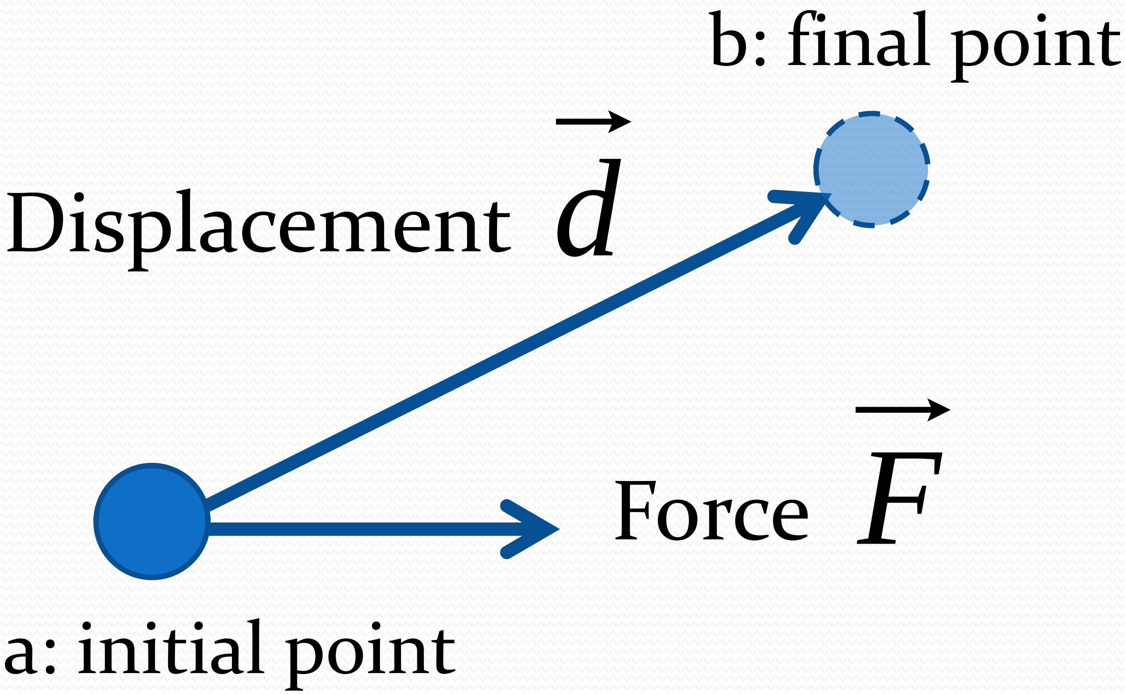
Chapter 17

Electric Potential

Force and Work: Reminder



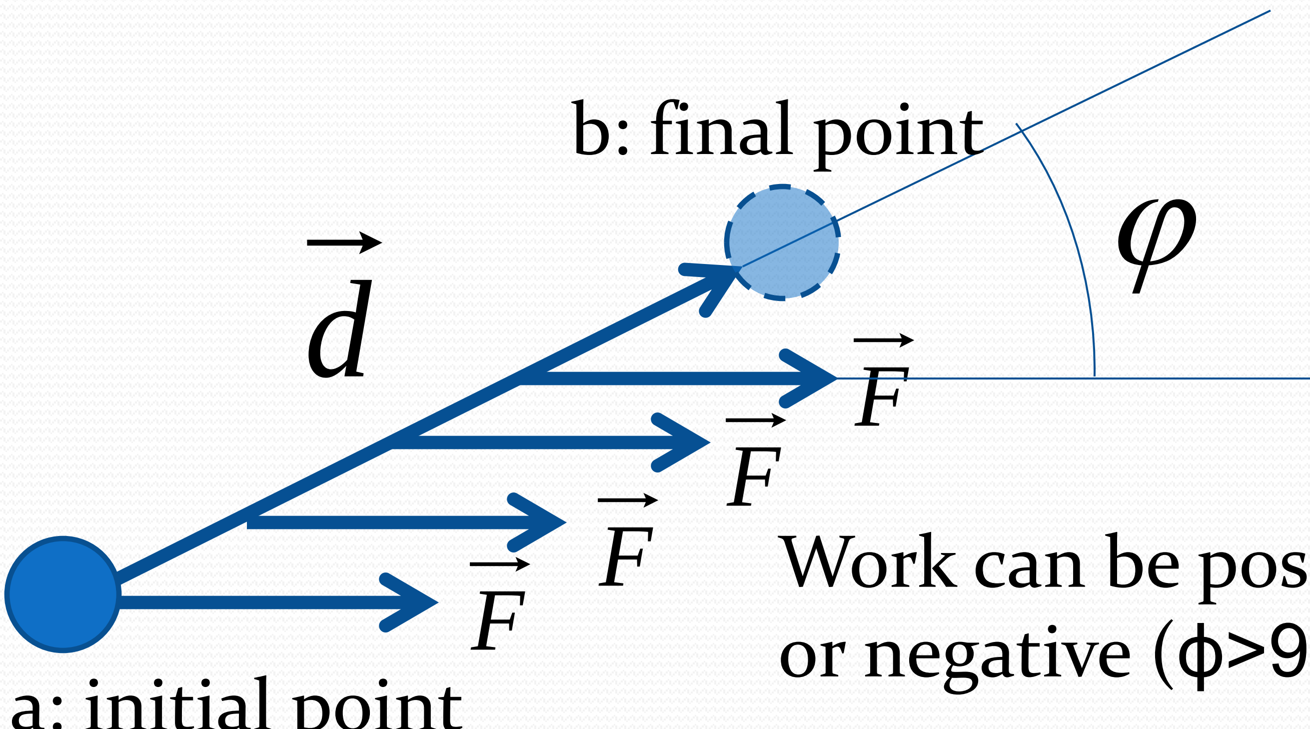
Reminder from Mechanics:

➢ if there is a force acting on an object (e.g. electric force), this force may do some work when the object moves

Force and Work

$$W_{a \to b} = Fd\cos\varphi$$

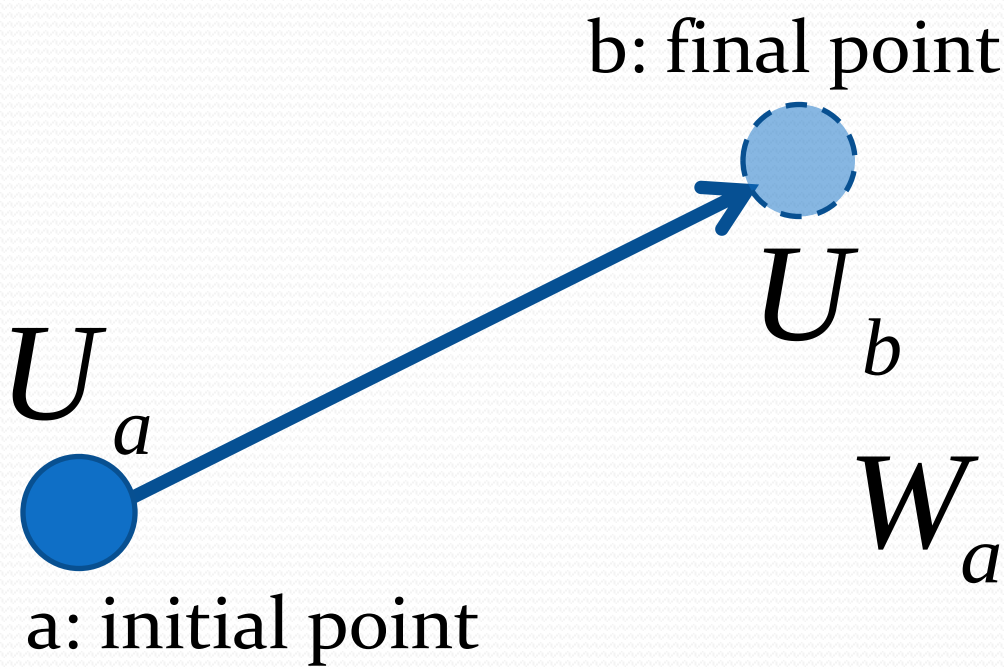
that's why direction matters



Work can be positive ($\phi<90°$) or negative ($\phi>90°$)

Conservative Force

- If a force is conservative, there is something called potential energy U which changes from point to point



Work does not depend on the path!

$$W_{a\to b} = U_a - U_b$$

Work-Energy Theorem

- Change in kinetic energy = work

$$K_b - K_a = W_{a \to b}$$

$$K_b - K_a = U_a - U_b$$

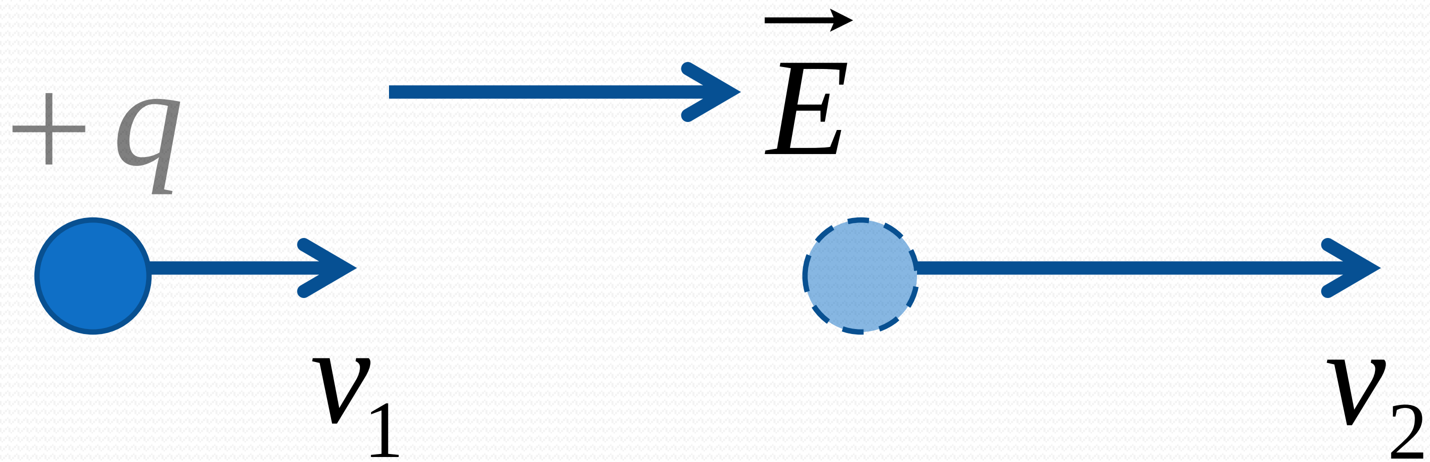
$$K_a + U_a = K_b + U_b$$

total energy doesn't change

Electric Potential Energy

- If charge q is placed in electric field E, the force acting on it is $\vec{F} = q\vec{E}$

$$U_a - U_b = W_{a\to b} = qEd\cos\varphi$$



Electric Potential

- Force → Field = Force divided by probe charge q
  - Field is independent of q
- Potential energy → Potential = Potential energy divided by probe charge q
  - Potential is independent of q

$$V = \frac{U}{q}$$

$[V]$ = Volt 1 V = 1 J / C

watch signs!

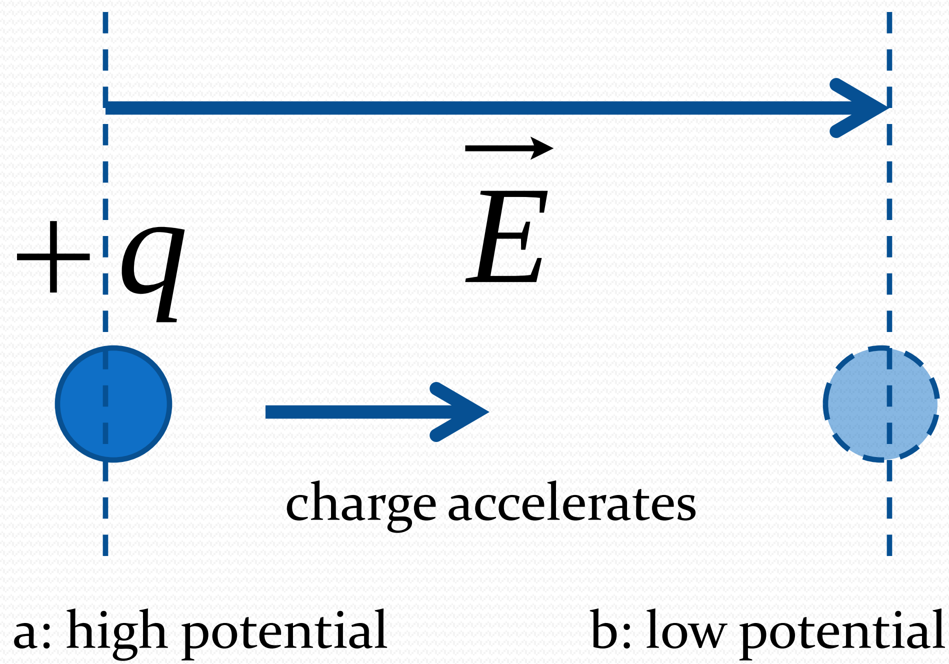
Electric Potential Difference

- Like potential energy, potential itself is meaningless, what makes sense is the potential difference

$$V_{ba} = -\frac{W_{a\to b}}{q}$$

potential difference ("voltage") between points a and b

$$V_{ba} = V_b - V_a$$



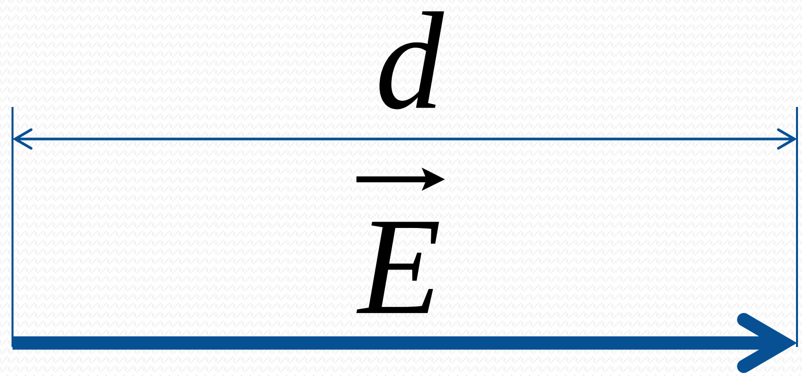
Relation between Electric Potential Difference and Electric Field

$$W_{a\rightarrow b} = qEd\cancel{\cos\varphi}$$

consider a probe charge moving parallel to electric field lines

$$V_{ba} = -\frac{W_{a\rightarrow b}}{q}$$

$$\boxed{V_{ba} = -Ed}$$



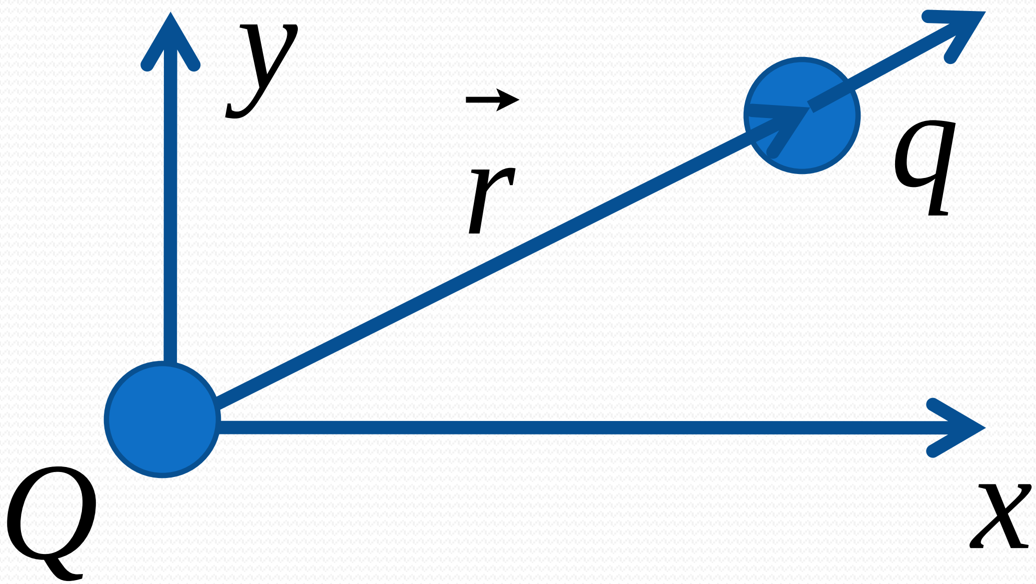
a: high potential

b: low potential

Field of Point Charge

- Charge produces electric field which can act on another charge

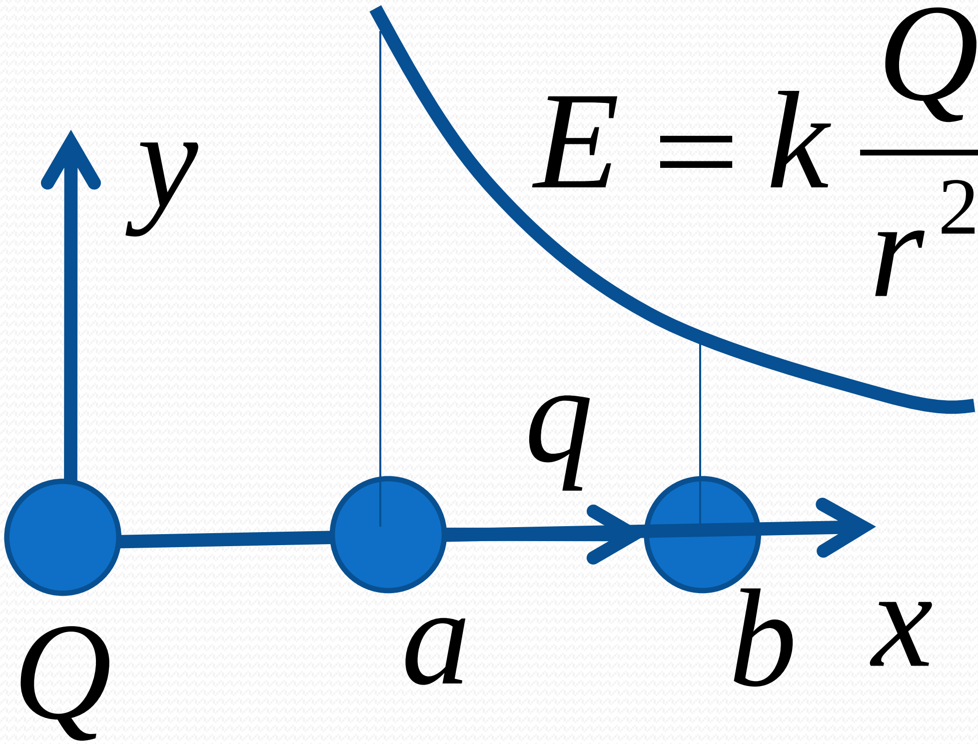
there is potential energy due to interaction of two charges



•Field is non-uniform
•Field is not constant

$$E = k\frac{Q}{r^2}$$

Work in Field of Point Charge



$$E = k\frac{Q}{r^2}$$

ϕ=0, OK

$$W = qEd\cos\varphi$$

E varies with *x*, not OK !

Need calculus to compute result

Work in Field of Point Charge

$$W_{a\to b} = kQq\left(\frac{1}{a} - \frac{1}{b}\right) = k\frac{Qq}{a} - k\frac{Qq}{b}$$

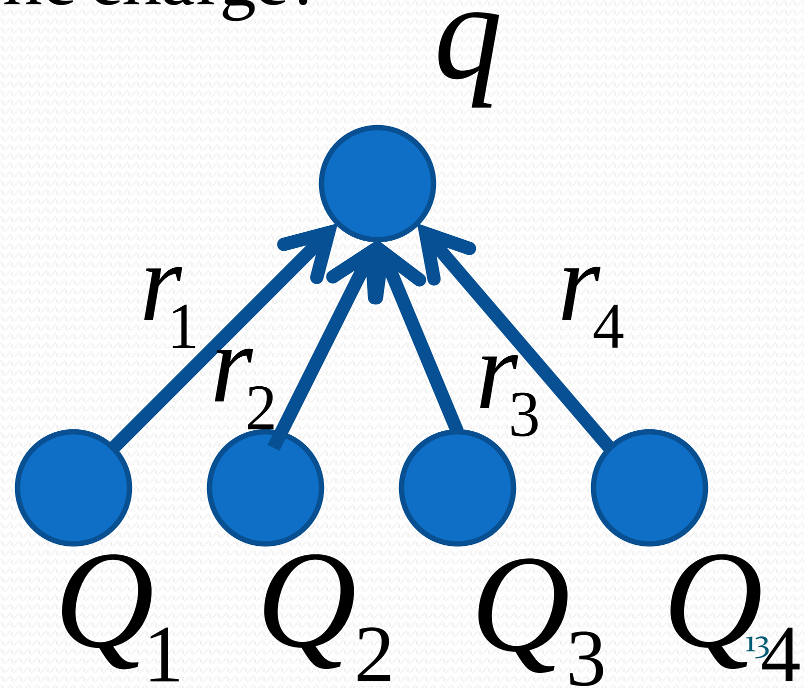
$$U = k\frac{Qq}{r}$$

Formula valid for any combination of q, q' signs

More about potential energy

- U is always defined w.r.t. U at some point chosen by convention (only U_a-U_b has physical meaning)
  - our choice: U=0 at r=∞
- What if there are more than one charge?

$$U = kq\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots\right)$$



Potential of a Point Charge

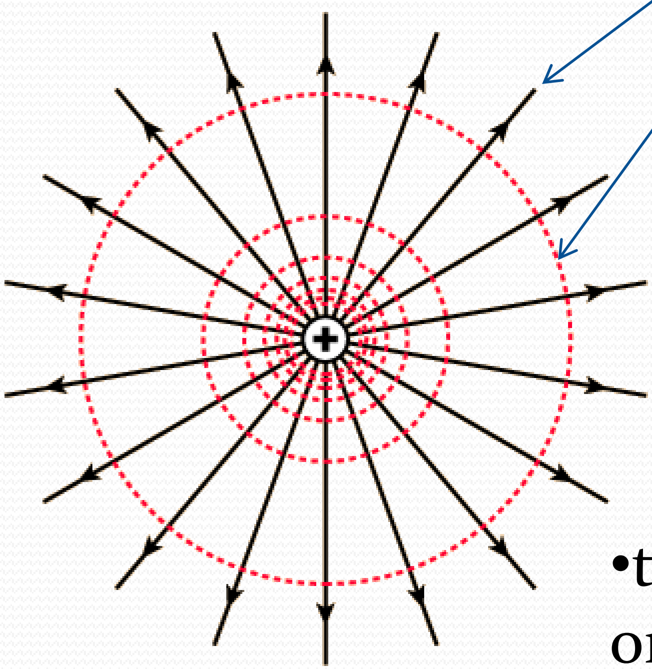
$$V = \frac{U}{q} = k\frac{Q}{r}$$

•Both potential and electric field are independent from test charge
•Potential is a scalar, field is a vector

For many point charges:

$$V = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots\right)$$

Equipotential Surfaces



electric field lines
equipotential surfaces

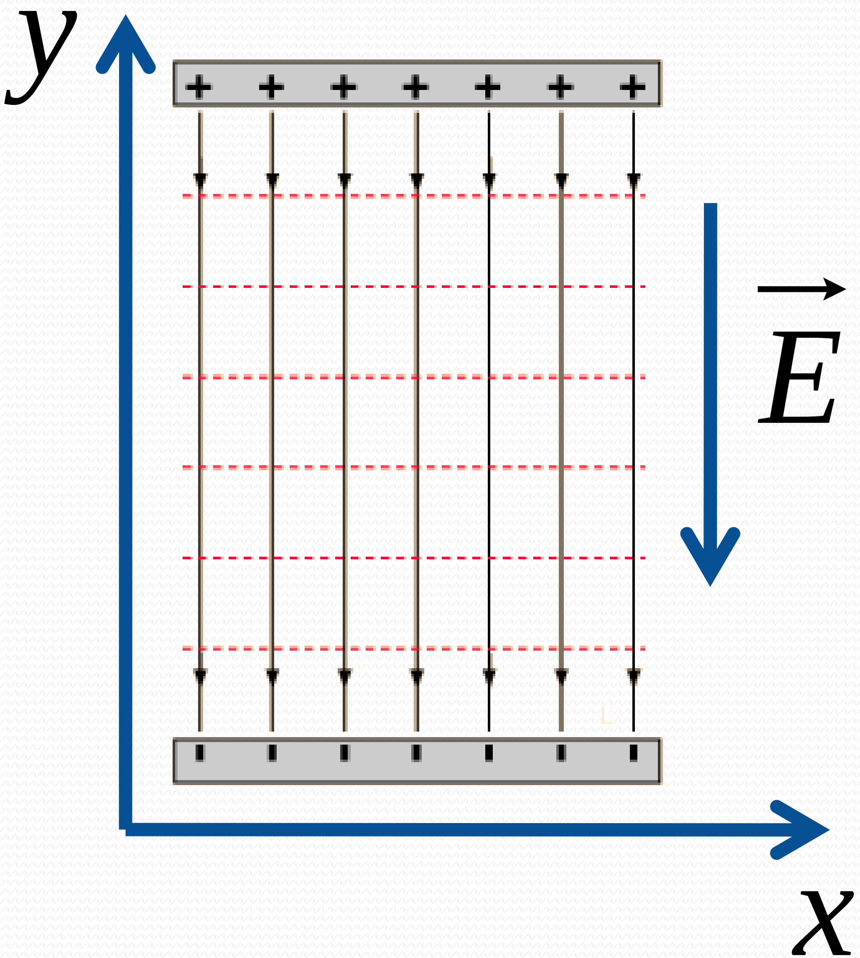
electric field line: imaginary line which in each point is tangent to electric field vector $\vec{E}$

equipotential surface: potential is the same at every point

•two equipotential surfaces never touch or intersect

•equipotential surfaces are mutually perpendicular to field lines

Uniform Field



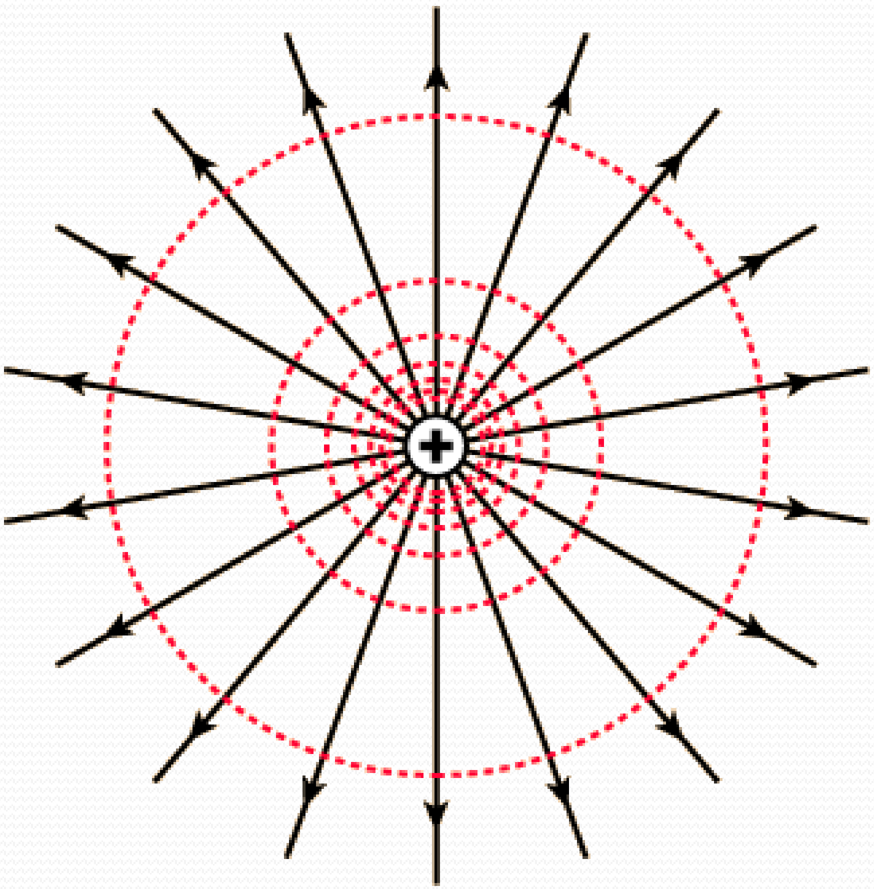
$$V = \frac{U}{q} = Ey$$

$$V = const \Rightarrow$$

$$y = const$$

equipotential surfaces =
planes perpendicular to y axis

Point Charge



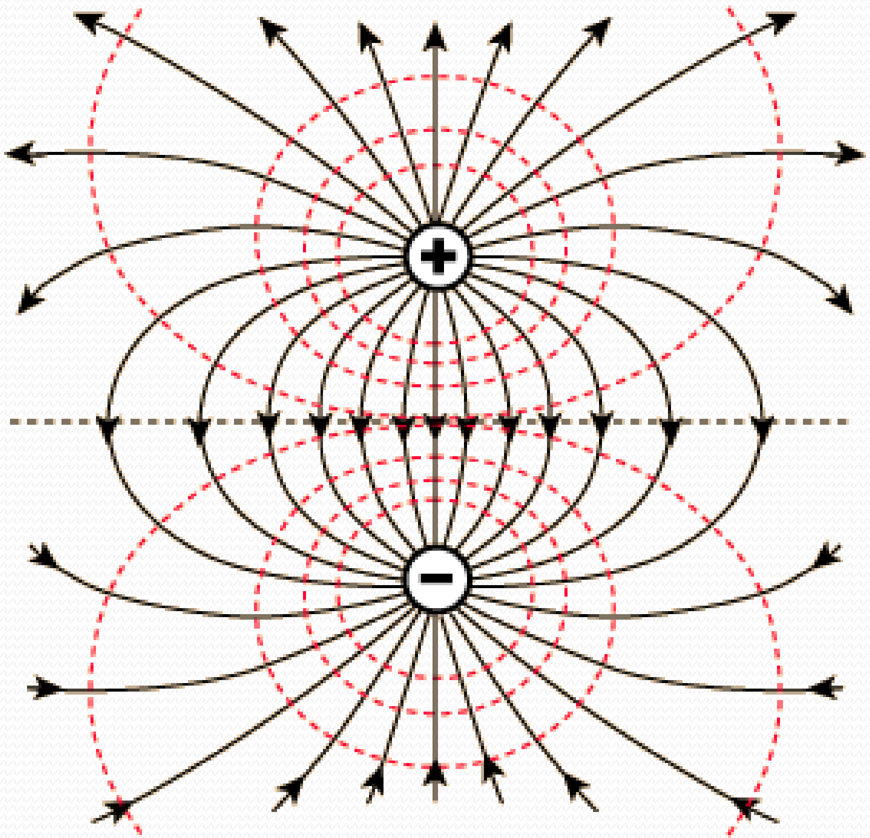
$$V = k\frac{Q}{r}$$

$$V = const \Rightarrow$$

$$r = const$$

equipotential surfaces = spheres with centers at origin

Dipole



If charges +q and −q are placed at y=+a and y=−a then plane y=0 is an equipotential surface

Proof: $V = k\left(\frac{Q}{r_1} - \frac{Q}{r_2}\right)$

at y=0 $r_1=r_2$, so V=0

Potential Gradient

- The magnitude of the electric field at any point on an equipotential surface = rate of change of potential

$$E = -\frac{\Delta V}{\Delta s}$$

electric field is a vector, so what about direction?

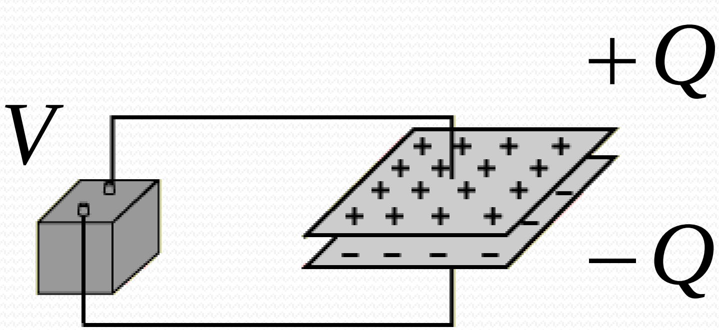
$$\Delta s = (\Delta x, \Delta y, \Delta z)$$

$$\vec{E} = \left(-\frac{\Delta V}{\Delta x}, -\frac{\Delta V}{\Delta y}, -\frac{\Delta V}{\Delta z} \right)$$

as a point moves along electric field, potential decreases

Capacitor and capacitance

- Capacitor = device which stores electric charge
  - Capacitor ≠ battery!
- Capacitor = two conductors separated by an insulator
  - generally, conductor 1 has charge q_1, and conductor 2 has charge q_2, but we'll always assume $q_1=Q=-q_2$, so the net charge is 0
- Capacitance C = capacitor's ability to store charge



$$C = \frac{Q}{V}$$

[C] = Farad, 1 F = 1 C / V

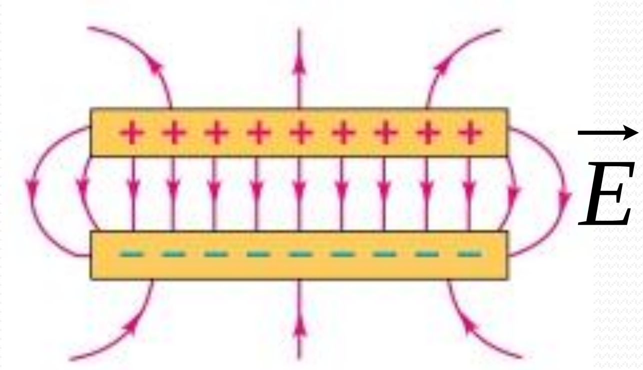
Parallel-Plate Capacitor

$$\sigma = \frac{Q}{A}$$ surface charge density

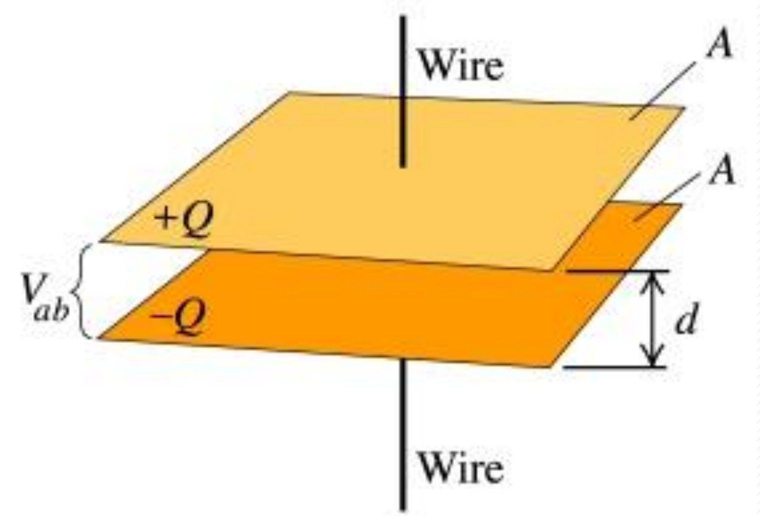
$$E = \frac{\sigma}{\varepsilon_0}$$ this can be proved using Gauss' law

$$V = Ed$$

$$C = \frac{Q}{V}$$



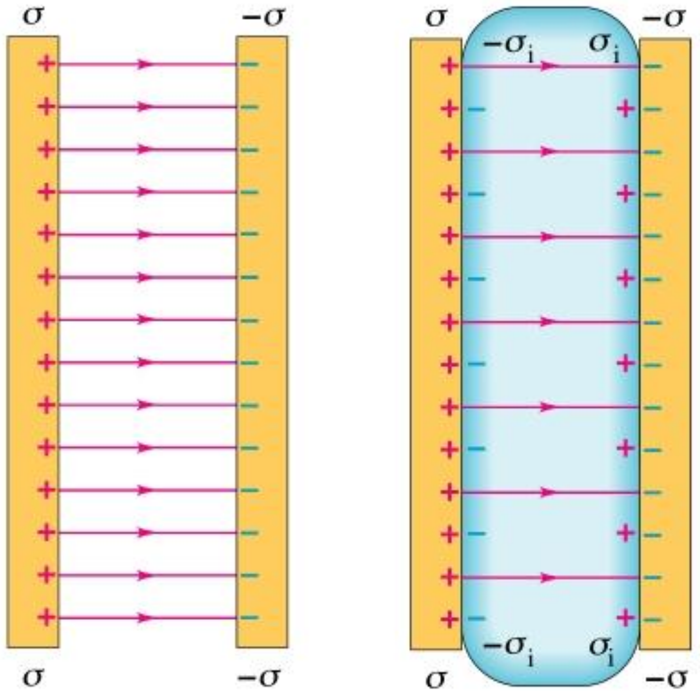
$$C = \varepsilon_0 \frac{A}{d}$$



$\varepsilon_0 = 1/4\pi k = 8.854 \times 10^{-12}\ C^2/N \cdot m^2$

Dielectrics

- If there is some material between the capacitor plates, the capacitance increases due to polarization



$$K = \frac{C}{C_0}$$

$$V = \frac{V_0}{K}$$

$$E = \frac{E_0}{K}$$

K = dielectric constant (a pure number depending on material)

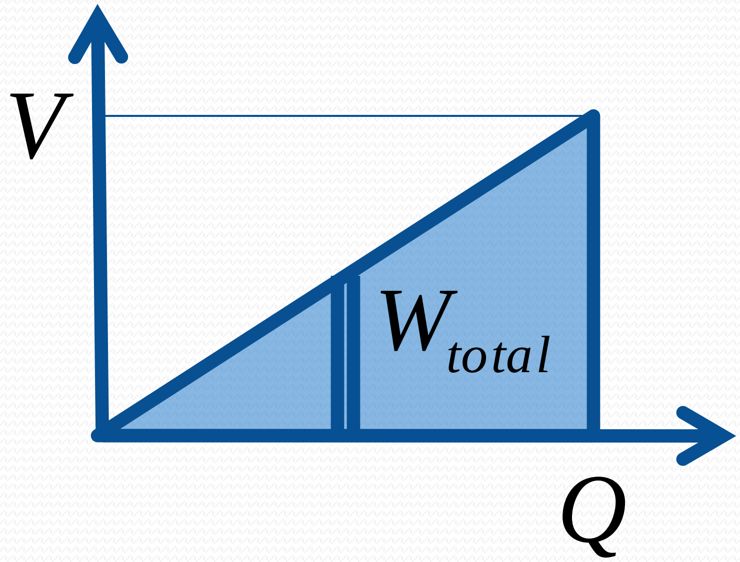
If we insert material inside capacitor charged with charge Q, the voltage decreases by K

Electric Field Energy

- Potential energy of a capacitor = work needed to charge it
  - If there is already charge Q and we want to add more charge ΔQ then we need to do work ΔW=VΔQ

$$\Delta W = V\Delta Q = \frac{Q\Delta Q}{C}$$

$$U = W_{total} = \frac{V}{2}Q = \frac{Q^2}{2C} = \frac{CV^2}{2}$$



Electric Field Energy Density

- Energy density u = energy U per unit volume v

$$v = Ad$$

$$u = \frac{U}{v} = \frac{CV^2}{2Ad}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$V = Ed$$

$$u = \frac{\varepsilon_0 E^2}{2}$$

there is nothing related to a capacitor in this formula – it's also valid for a field in vacuum!