Chapter 19

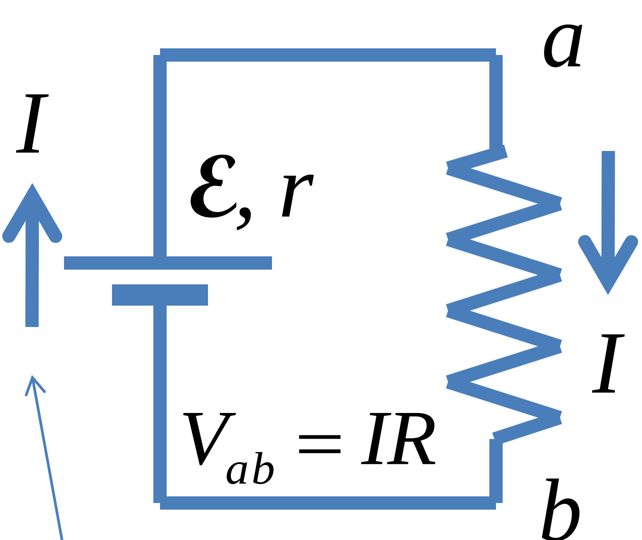
DC Circuits

Electromotive Force

= the influence that moves charge from lower to higher potential (against electric field)

emf is not a force ☺ It's energy per charge – like potential

emf origin: chemical (battery), magnetic field (generator),…



current inside the battery: from – to + due to emf

current through resistance: due to potential difference

ideal emf source: $V_{ab} = \mathcal{E}$

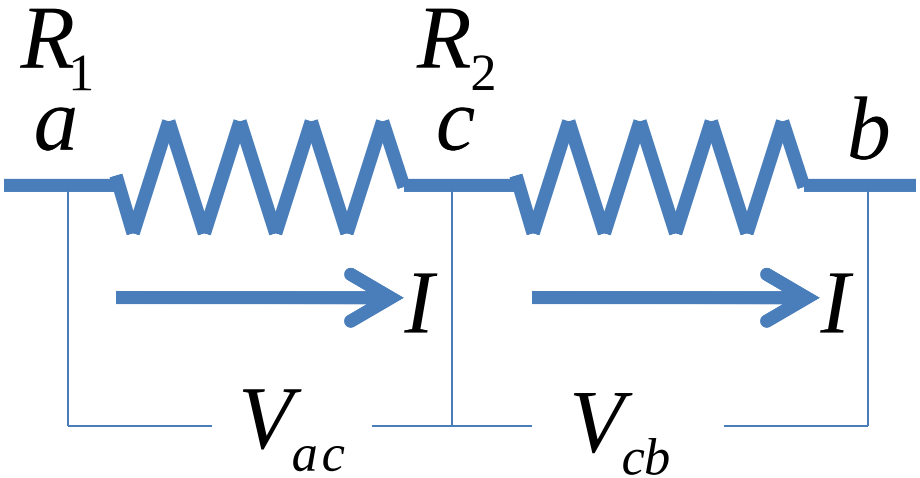
real emf source: $V_{ab} = \mathcal{E} - Ir$

terminal voltage

internal resistance

$$I = \frac{\mathcal{E}}{R + r}$$

Resistors in Series



$$V_{ac} = IR_1$$

$$V_{cb} = IR_2$$

$$V_{ab} = IR_{eq}$$

current is the same, potential difference adds up (no charge accumulation)

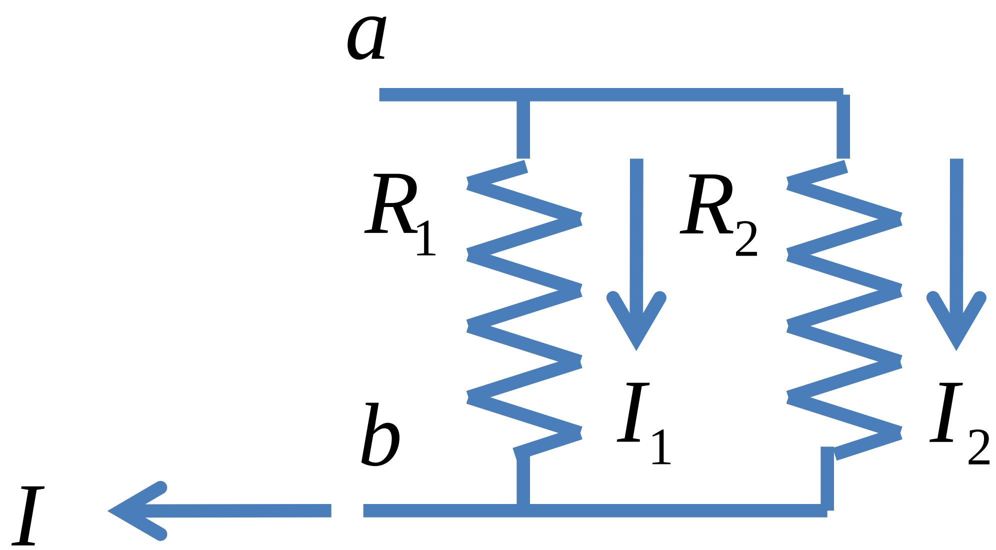
$$V_{ab} = V_{ac} + V_{cb}$$

$$R_{eq} = R_1 + R_2$$

more than two resistors in series:

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

Resistors in Parallel



$$V_{ab} = I_1 R_1$$

$$V_{ab} = I_2 R_2$$

$$V_{ab} = I R_{eq}$$

potential difference is the same, current adds up (no charge accumulation)

$$I = I_1 + I_2$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

more than two resistors in parallel:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Kirchhoff's Rules

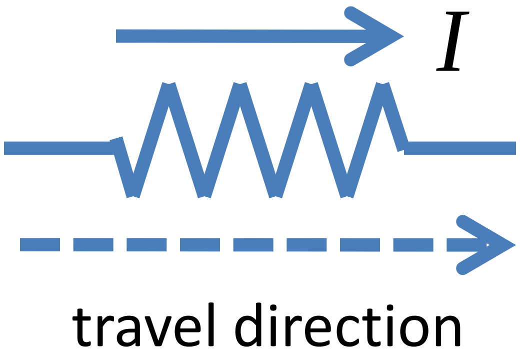
- use them when the circuit can't be reduced to series/parallel combinations

Point rule: algebraic sum of all currents into a point is 0

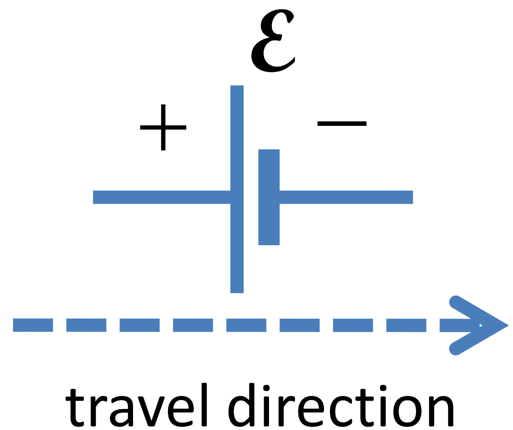
Loop rule: algebraic sum of all potential differences in a loop is 0

- label currents
- for each junction, write down the sum of currents
- for each loop, write down the sum of potential differences

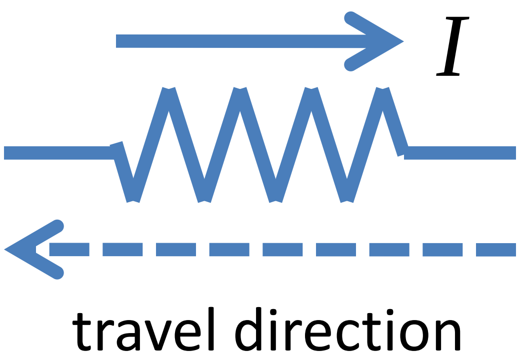
Rules for Loops



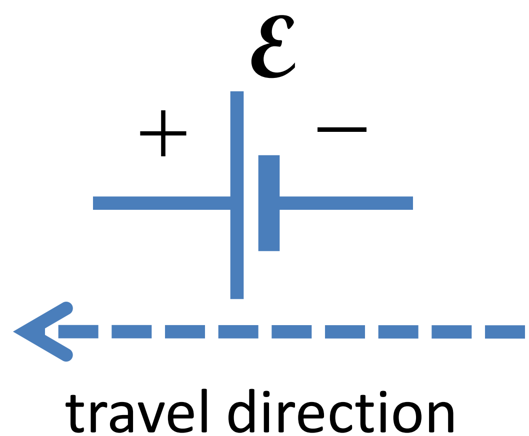
$-IR$



$-\mathcal{E}$

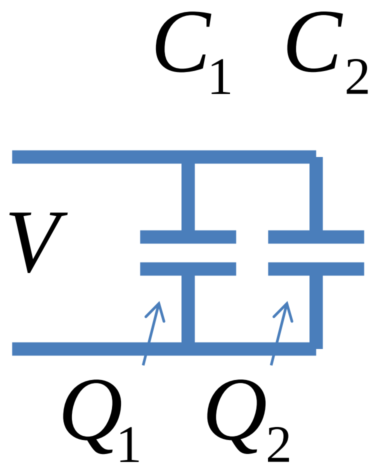


$+IR$



$+\mathcal{E}$

Capacitors in Parallel



$$Q_1 = C_1 V$$

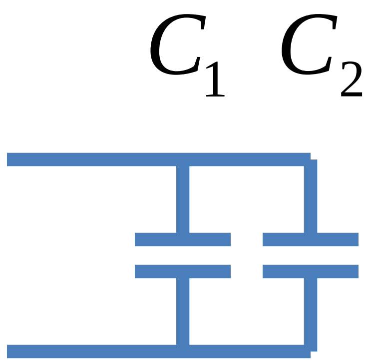
$$Q_2 = C_2 V$$

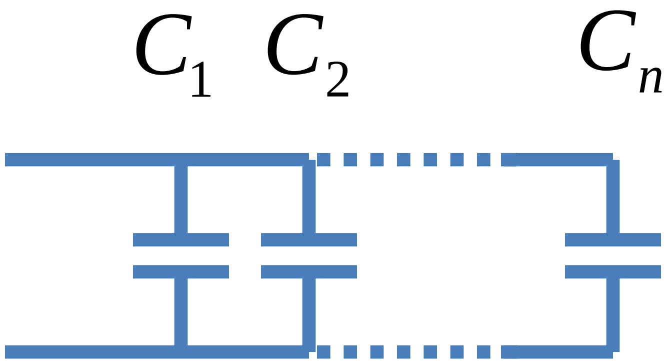
voltage on both capacitors is the same!

$$Q = Q_1 + Q_2 = V(C_1 + C_2)$$

$$C_{eq} = \frac{Q}{V}$$

Capacitors in Parallel: formula

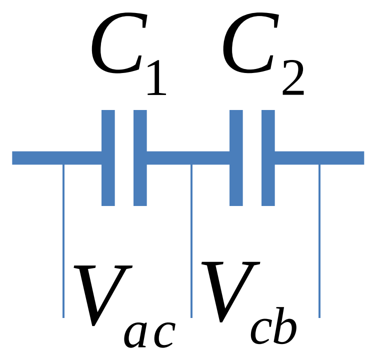




$$C_{eq} = C_1 + C_2$$

$$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Capacitors in Series



$$V_{ac} = \frac{Q}{C_1}$$

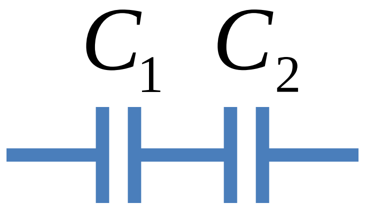
$$V_{cb} = \frac{Q}{C_2}$$

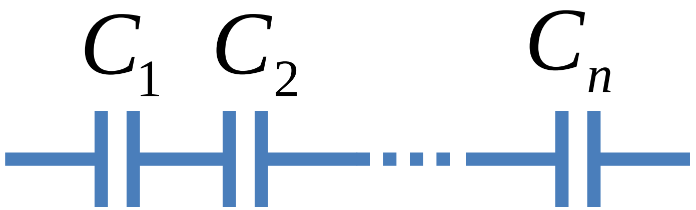
charge on both capacitors is the same! (conservation of charge)

$$V_{ab} = V_{ac} + V_{cb} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$C_{eq} = \frac{Q}{V_{ab}}$$

Capacitors in Series: formula

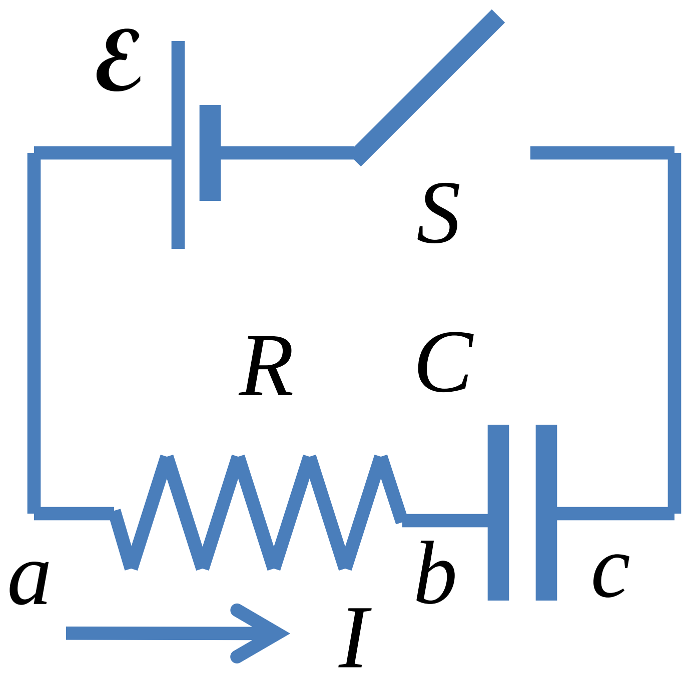




$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}}$$

Resistance-Capacitance Circuits



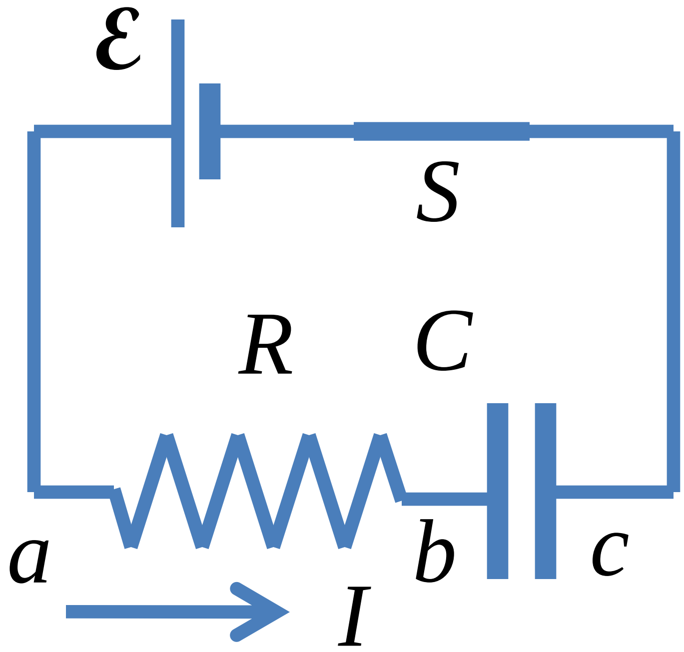
Switch open

$$I = 0$$

$$V_{ab} = V_{bc} = 0$$

$$Q = 0$$

Resistance-Capacitance Circuits



Switch closed $(t = 0)$

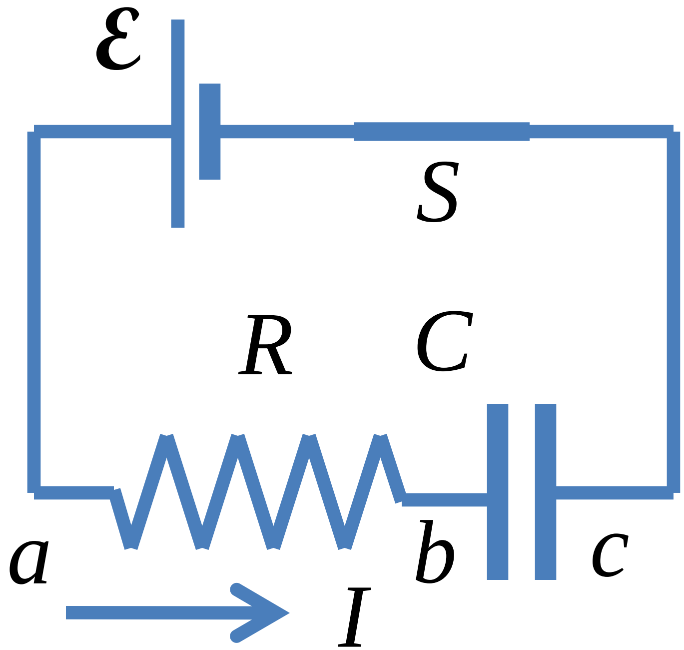
$Q = 0$ C needs time to charge

$$V_{bc} = \frac{Q}{C} = 0$$

$$V_{ab} = IR \neq 0$$

$\mathcal{E} = IR$ Kirchhoff's rule

Resistance-Capacitance Circuits



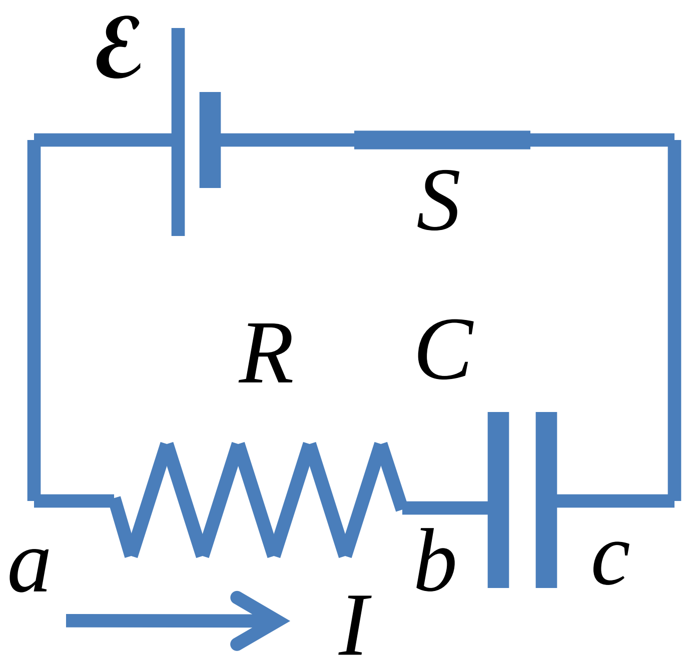
Switch closed $(t > 0)$

$$Q > 0$$

$$V_{bc} = \frac{Q}{C} > 0$$

$$\mathcal{E} = IR + \frac{Q}{C} \qquad (I < I_0)$$

Resistance-Capacitance Circuits



Switch closed $(t = \infty)$

$$I = 0$$

no current in static condition (C plates insulated!)

$$V_{ab} = 0$$

$$\mathcal{E} = \frac{Q}{C}$$

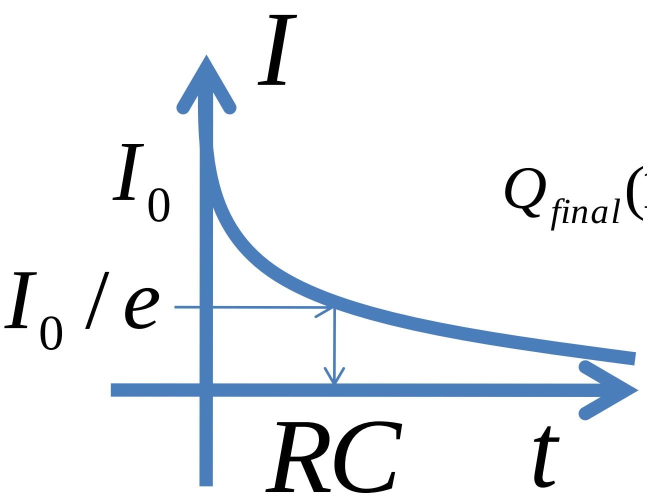
final charge doesn't depend on R

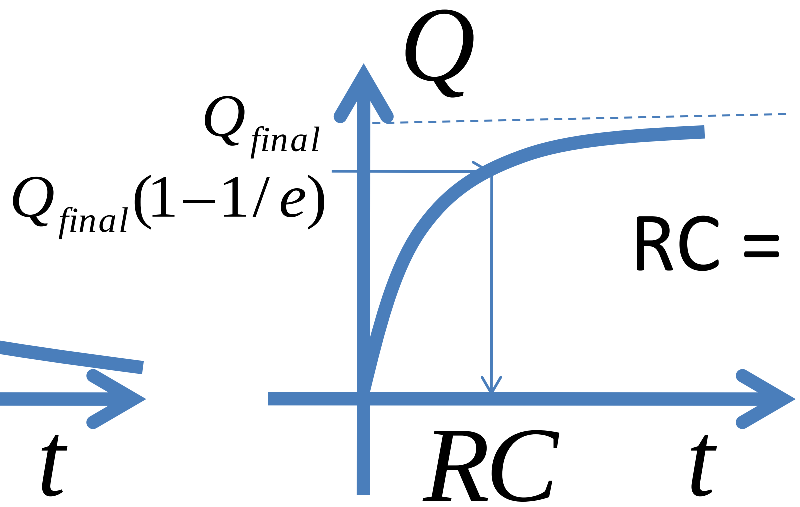
Current and Charge vs time

$$\mathcal{E} = IR + \frac{Q}{C} \quad \text{where} \quad I = \frac{\Delta Q}{\Delta t}$$ – need calculus to solve it

Answer: $I = I_0 e^{-t/RC}, Q = Q_{final}(1 - e^{-t/RC})$

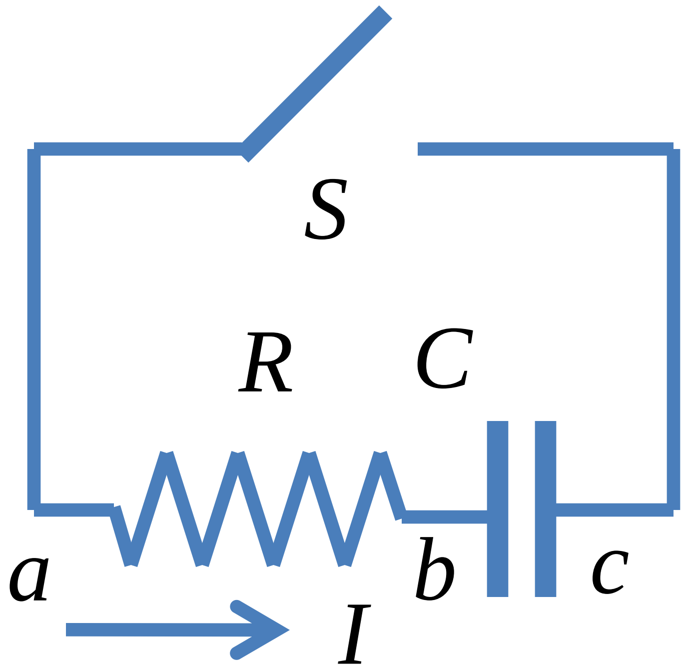
$e = 2.71828\ldots$





RC = relaxation time

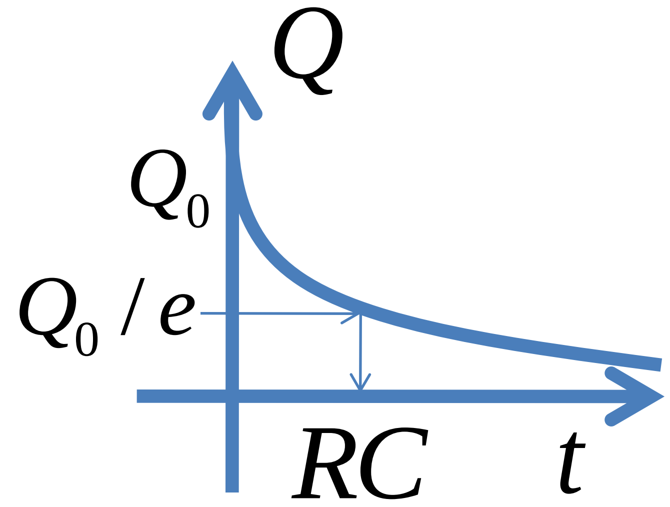
Capacitor Discharge



$$t = 0$$

$$Q = CV_0$$

$$I = -\frac{V_{bc}}{R}$$



$$I = I_0 e^{-t/RC}, Q = Q_0 e^{-t/RC}$$