

Chapter 19

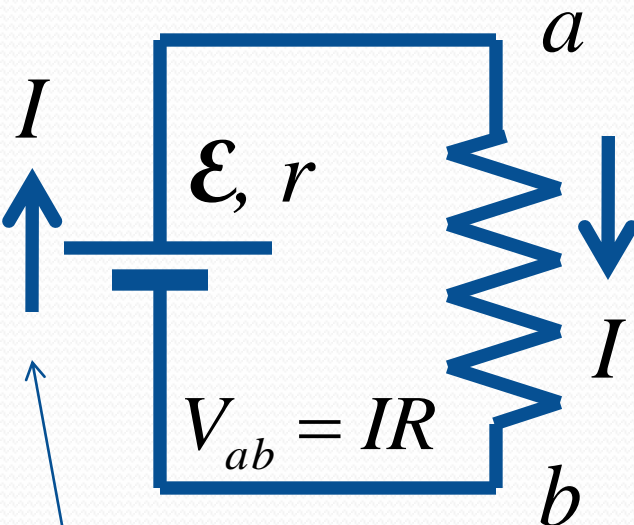
DC Circuits

Electromotive Force

= the influence that moves charge from lower to higher potential (against electric field)

emf is not a force ☺ It's energy per charge – like potential

emf origin: chemical (battery), magnetic field (generator),...



ideal emf source: $V_{ab} = \mathcal{E}$

real emf source: $V_{ab} = \mathcal{E} - Ir$

terminal voltage

internal resistance

$$I = \frac{\mathcal{E}}{R + r}$$

current through resistance:
due to potential difference

current inside the battery:
from - to + due to emf

Resistors in Series

$$V_{ac} = IR_1$$

$$V_{cb} = IR_2$$

$$V_{ab} = IR_{eq}$$

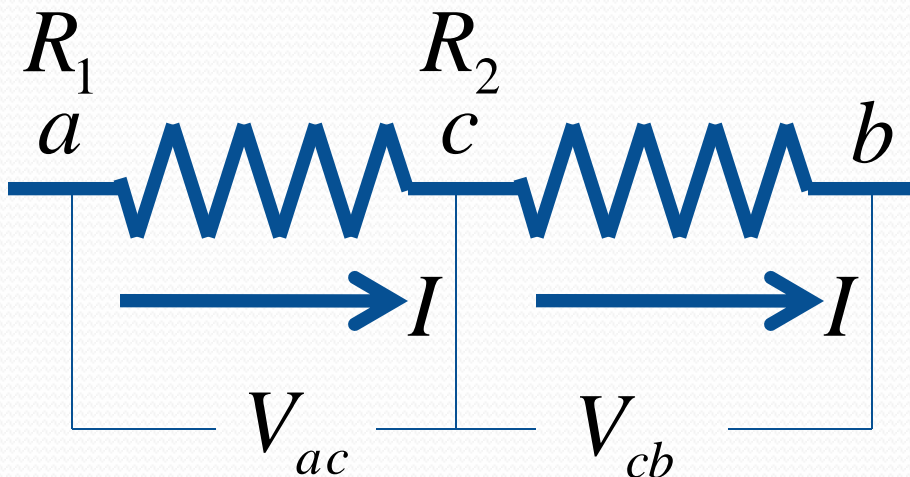
current is the same, potential difference adds up
(no charge accumulation)

$$V_{ab} = V_{ac} + V_{cb}$$

more than two resistors in series:

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + \dots + R_n$$



Resistors in Parallel

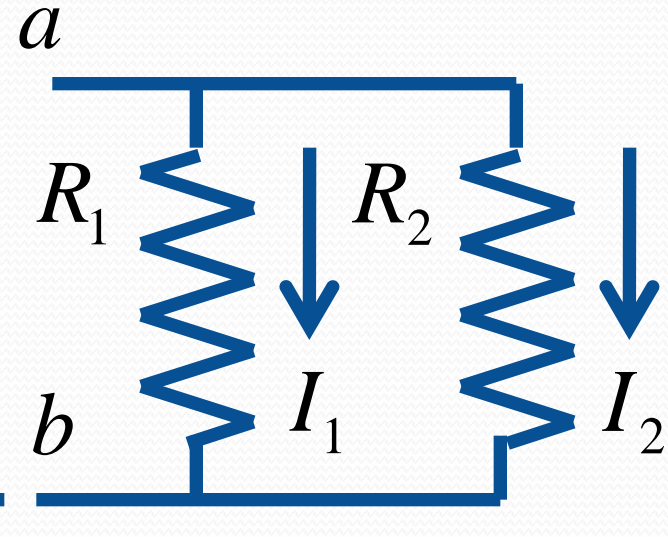
$$V_{ab} = I_1 R_1$$

$$V_{ab} = I_2 R_2$$

$$V_{ab} = I R_{eq}$$

$$I = I_1 + I_2$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



potential difference is the same, current adds up
(no charge accumulation)

more than two resistors in parallel:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

Kirchhoff's Rules

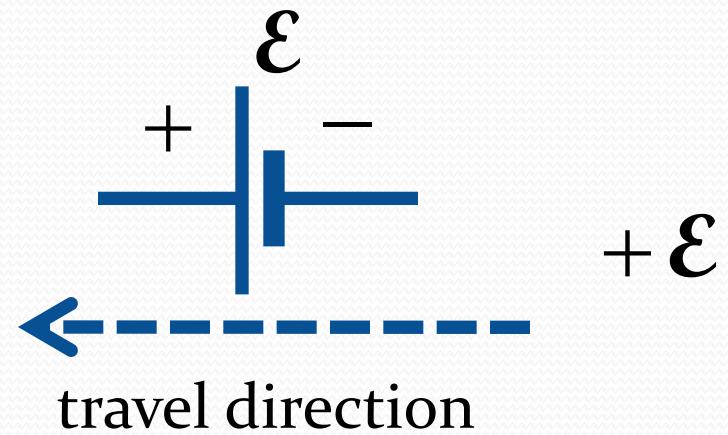
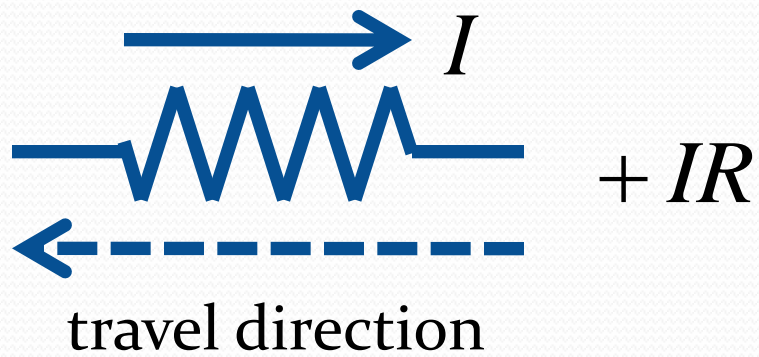
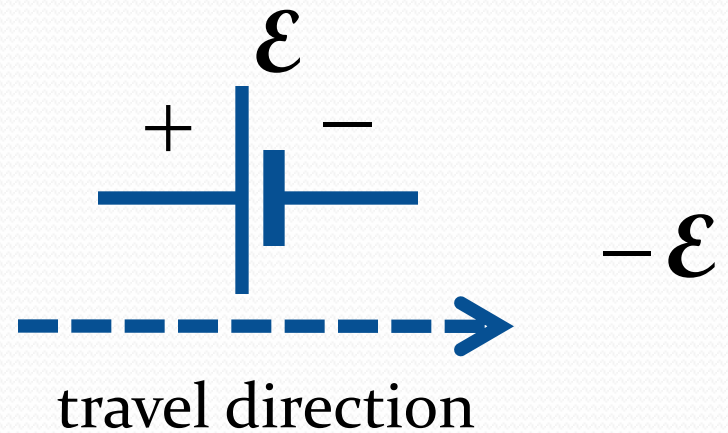
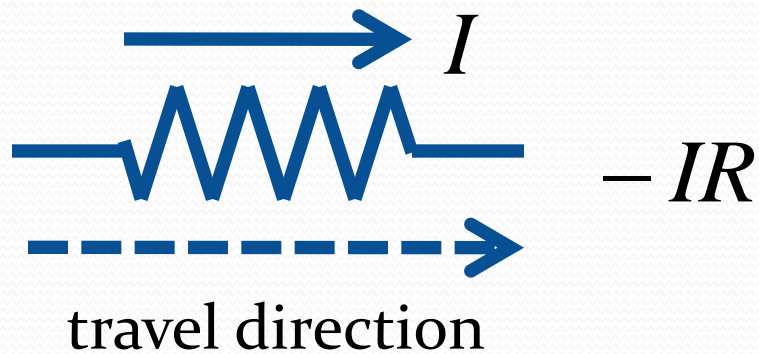
- use them when the circuit can't be reduced to series/parallel combinations

Point rule: algebraic sum of all currents into a point is 0

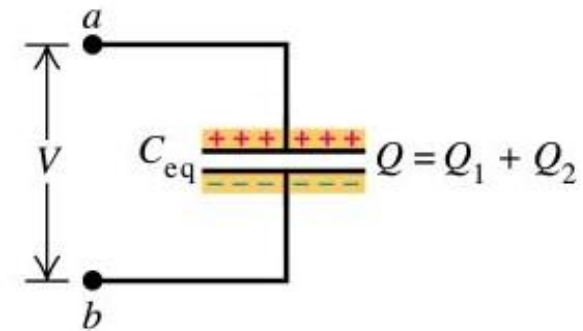
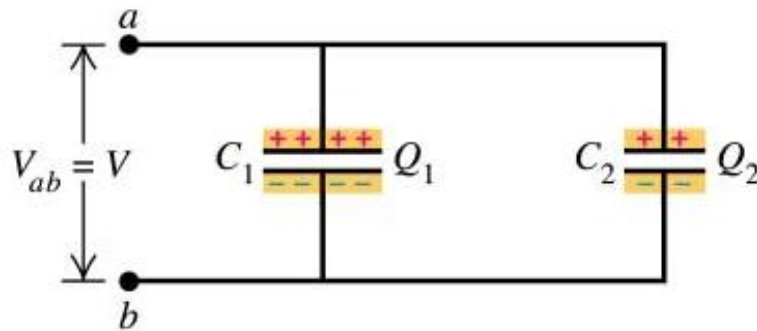
Loop rule: algebraic sum of all potential differences in a loop is 0

- label currents
- for each junction, write down the sum of currents
- for each loop, write down the sum of potential differences

Rules for Loops



Capacitors in Parallel



$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

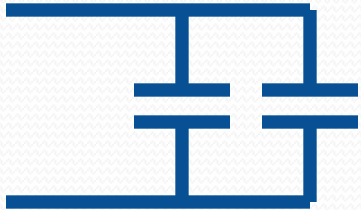
voltage on both capacitors is the same!

$$Q = Q_1 + Q_2 = V(C_1 + C_2)$$

$$C_{eq} = \frac{Q}{V}$$

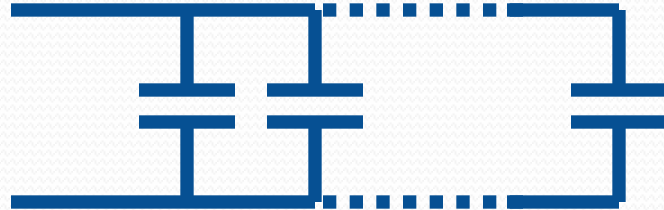
Capacitors in Parallel: formula

C_1 C_2



$$C_{eq} = C_1 + C_2$$

C_1 C_2 C_n



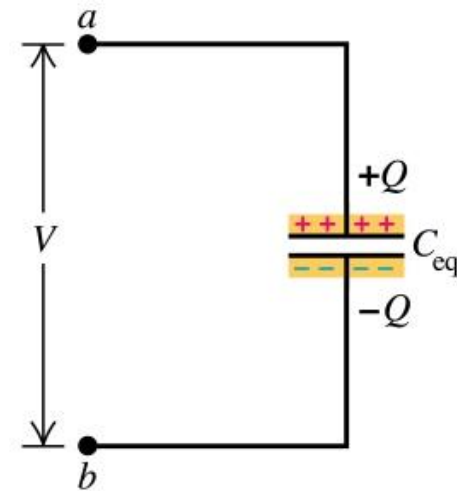
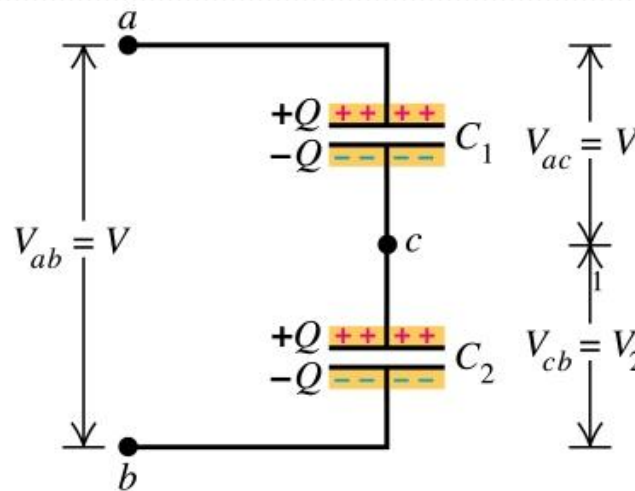
$$C_{eq} = C_1 + C_2 + \dots + C_n$$

Capacitors in Series

$$V_{ac} = \frac{Q}{C_1}$$

$$V_{cb} = \frac{Q}{C_2}$$

charge on both capacitors is the same! (conservation of charge)



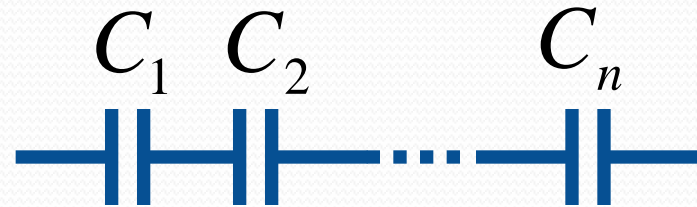
$$V_{ab} = V_{ac} + V_{cb} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$C_{eq} = \frac{Q}{V_{ab}}$$

Capacitors in Series: formula



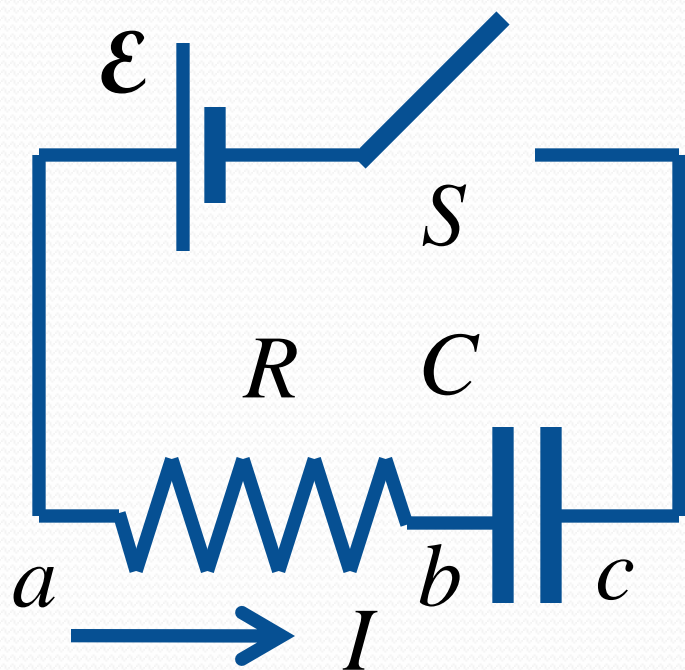
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$



$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

Resistance-Capacitance Circuits

Switch open



$$I = 0$$

$$V_{ab} = V_{bc} = 0$$

$$Q = 0$$

Resistance-Capacitance Circuits

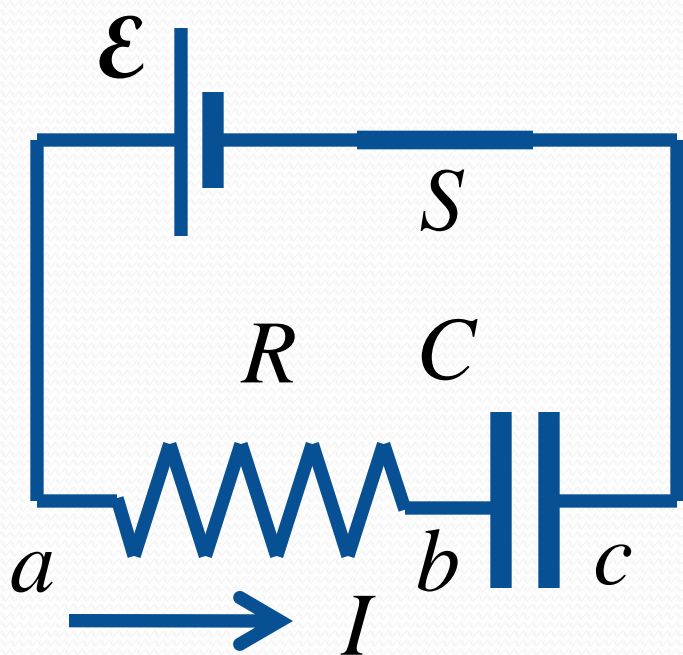
Switch closed ($t = 0$)

$Q = 0$ C needs time to charge

$$V_{bc} = \frac{Q}{C} = 0$$

$$V_{ab} = IR \neq 0$$

$$\mathcal{E} = IR \quad \text{Kirchhoff's rule}$$



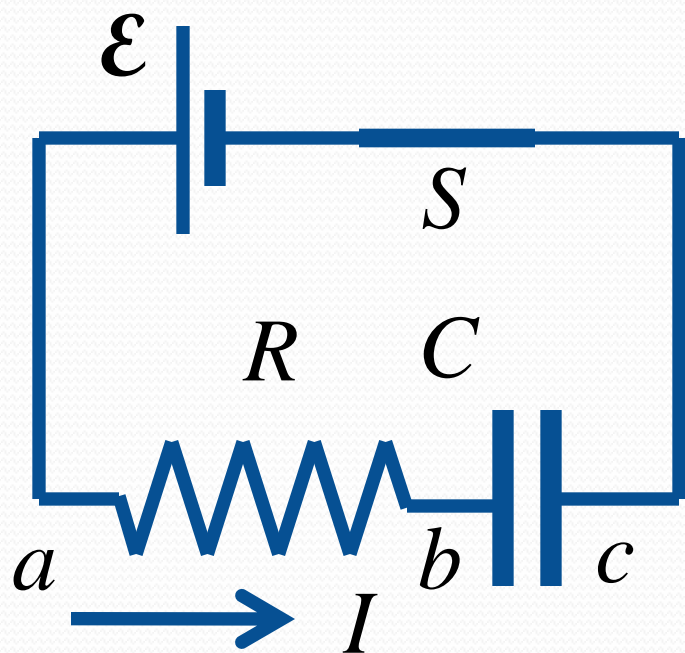
Resistance-Capacitance Circuits

Switch closed ($t > 0$)

$$Q > 0$$

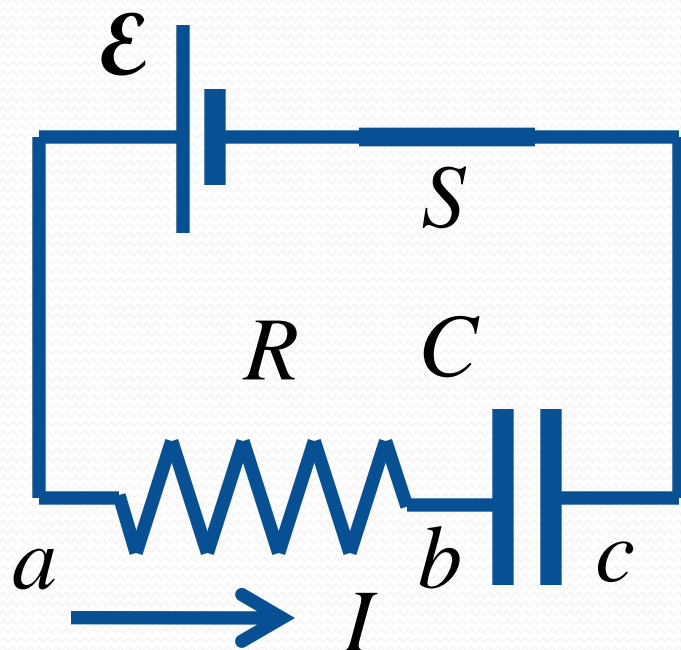
$$V_{bc} = \frac{Q}{C} > 0$$

$$\mathcal{E} = IR + \frac{Q}{C} \quad (I < I_0)$$



Resistance-Capacitance Circuits

Switch closed ($t = \infty$)



$$I = 0$$

$$V_{ab} = 0$$

$$\mathcal{E} = \frac{Q}{C}$$

no current in static condition (C plates insulated!)

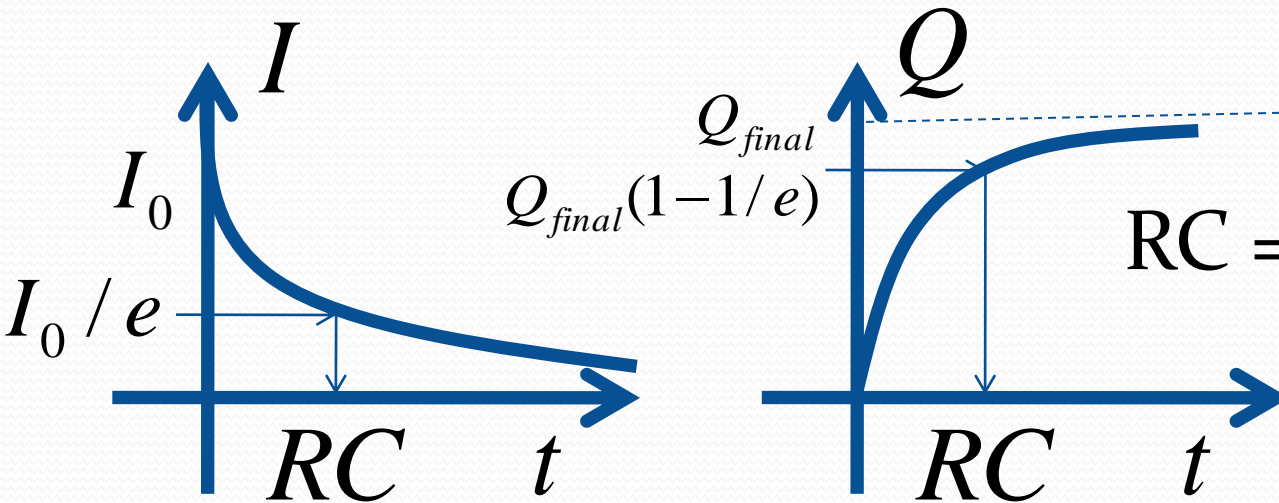
final charge doesn't depend on R

Current and Charge vs time

$$\mathcal{E} = IR + \frac{Q}{C} \quad \text{where} \quad I = \frac{\Delta Q}{\Delta t} \quad - \text{ need calculus to solve it}$$

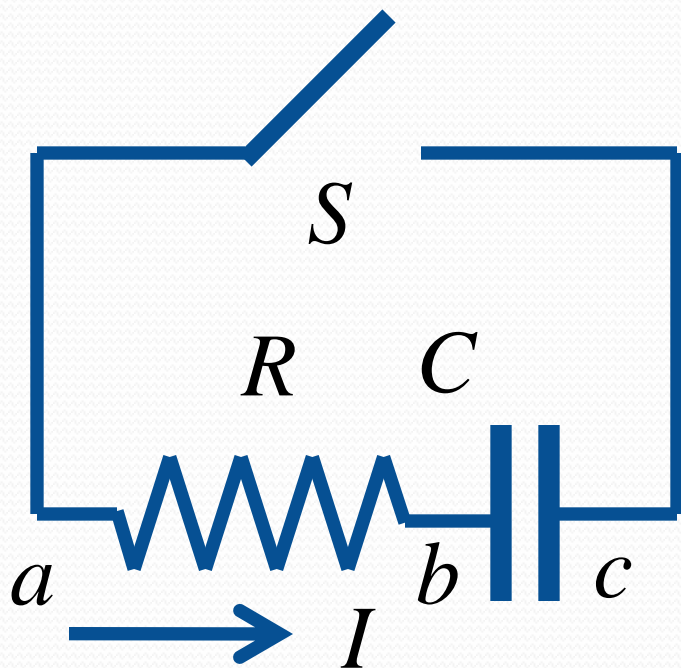
$$\text{Answer: } I = I_0 e^{-t/RC}, \quad Q = Q_{final} (1 - e^{-t/RC})$$

$$e = 2.71828\dots$$



$RC = \text{relaxation time}$

Capacitor Discharge



$$t = 0$$

$$Q = CV_0$$

$$I = -\frac{V_{bc}}{R}$$

$$I = I_0 e^{-t/RC}, Q = Q_0 e^{-t/RC}$$

