

# Chapter 21

## Electromagnetic Induction

# Induced EMF

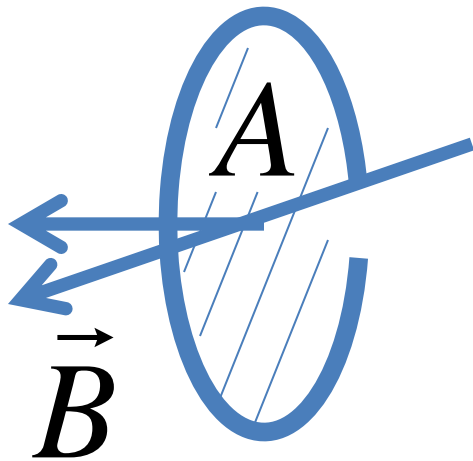
- We already know that moving charge (=current) causes magnetic field
- It also works the other way around: changing magnetic field (e.g. moving permanent magnet) causes current
  - it's called *induced current*
  - since there is a current, there is electromotive force called *induced emf*

# Induction Experiments

- Stationary permanent magnet does not induce current, the moving one does
- Stationary current in the first coil does not induce the current in the second coil, moving the coil in and out *or* switching the current on and off does

What matters is *change* of magnetic field

# Magnetic Flux



A is like a vector:  
you need to pick  
one of the two  
possible directions

$$\Delta\Phi = B\Delta A$$

if  $B$  is not perpendicular to surface then

$$\Delta\Phi = B \cos \varphi \Delta A$$

$$\Phi = \sum B \cos \varphi \Delta A$$

if  $B \cos \varphi$  is the same everywhere then

$$\Phi = BA \cos \varphi$$

# Faraday's Law

$$\mathcal{E} = - \frac{\Delta\Phi}{\Delta t}$$

rate of flux change

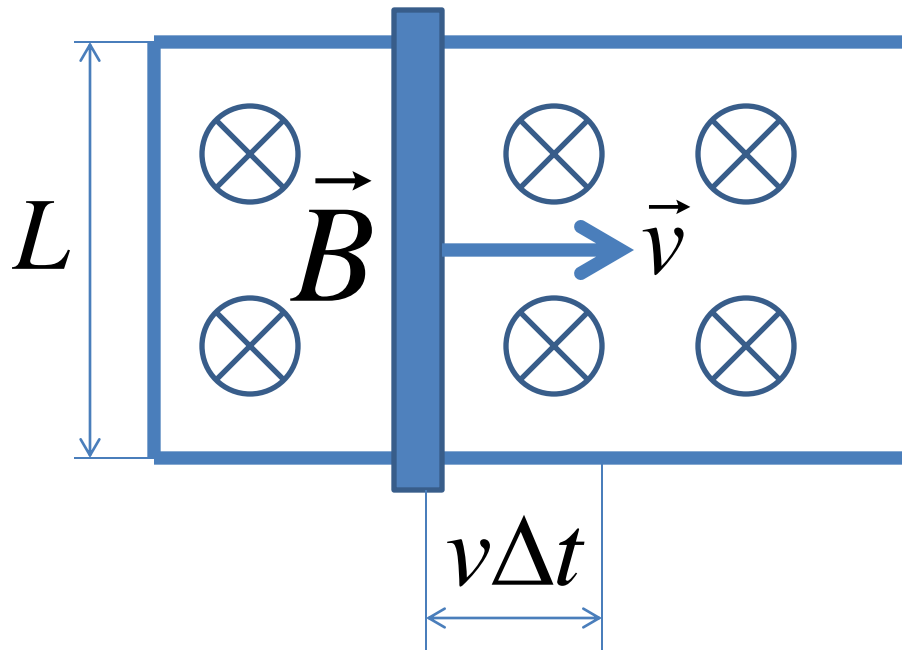
for a coil of N turns,  $\mathcal{E} = -N \frac{\Delta\Phi}{\Delta t}$

induced emf

$[\Phi] = \text{weber}, 1 \text{ Wb} = 1 \text{ T}\cdot\text{m}^2 = 1 \text{ V}\cdot\text{s}$

notice the “-” sign in the formula

# A Slide-Wire Generator



$$B = \text{const}$$

$$\Delta A = Lv\Delta t$$

$$\mathcal{E} = -\frac{B\Delta A}{\Delta t}$$

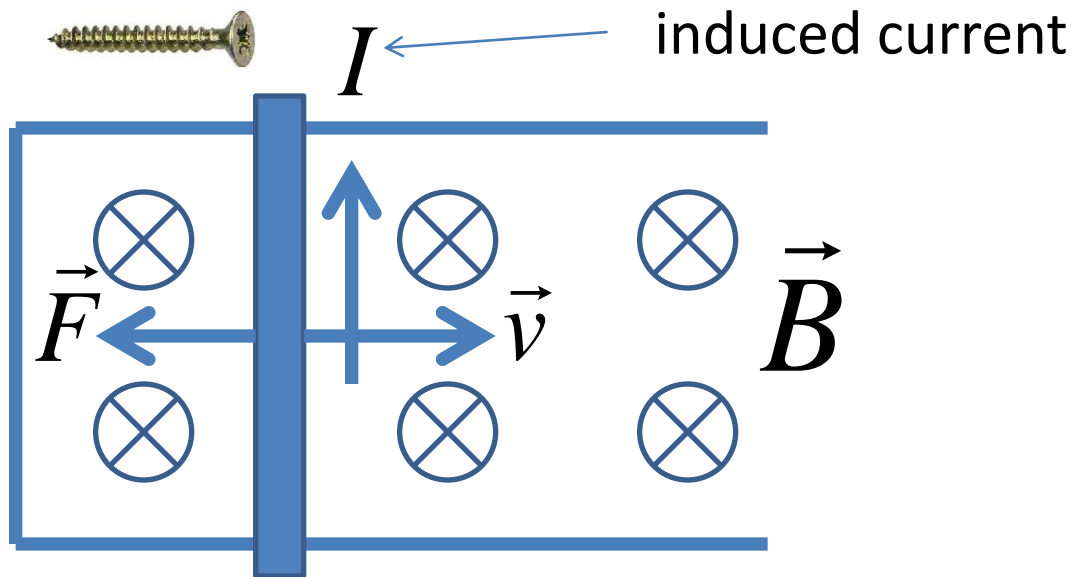
$$= -BLv$$

# Lenz's Law

- The direction of induced current is such as to oppose the direction of the phenomenon causing it

Lenz's law is a particular case of Le Chatelier's principle: in a stable equilibrium, any deviation would cause a force which drives the system back to the equilibrium (that's why it's stable!)

# Lenz's Law and a Slide-Wire rod



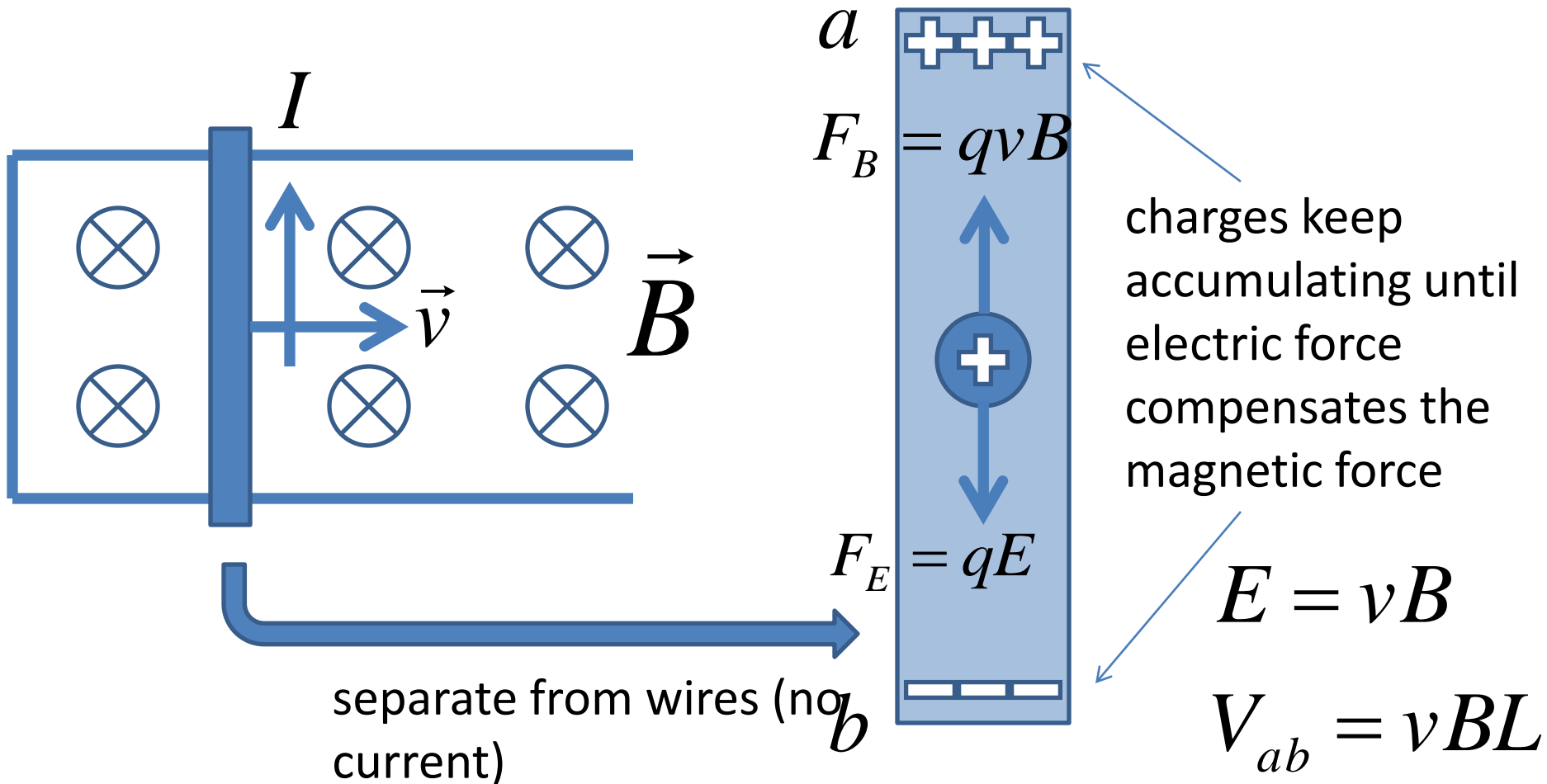
$\odot \vec{B}_{induced}$   
induced field is  
directed  
opposite to  $\vec{B}$

direction of induced current is such that force  $\vec{F}$  acting on it due to  $\vec{B}$  tries to slow down the moving rod

otherwise we would get infinite acceleration without external force



# Motional Electromotive Force



# Generator

- =device which converts mechanical energy to electricity

$$\varphi = \omega t$$

$$\Phi = AB \cos \omega t$$

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t} = \omega AB \sin \omega t$$

need calculus to derive it

# Mutual Inductance

- =coupling between two coils

$$\mathcal{E}_2 = N_2 \left| \frac{\Delta \Phi_2}{\Delta t} \right|$$

- when current through the main coil changes, the secondary coil gets induced emf
- the question is, how large is it?

$\Phi_2$  is proportional to the current in the main coil:

$$\Phi_2 = \frac{1}{N_2} M_{21} |i_1|$$

for convenience

$$\mathcal{E}_2 = M_{21} \left| \frac{\Delta i_1}{\Delta t} \right|$$

# Mutual Inductance

$$\mathcal{E}_2 = M_{21} \left| \frac{\Delta i_1}{\Delta t} \right| \quad \mathcal{E}_1 = M_{12} \left| \frac{\Delta i_2}{\Delta t} \right|$$

it turns out that

$$M = M_{12} = M_{21}$$

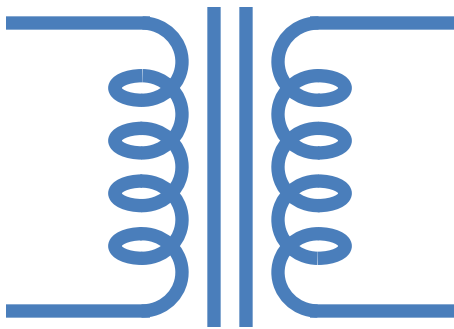
even for different  
coils

$$[M] = \text{henry}, 1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ V}\cdot\text{s/A} = 1 \text{ }\Omega\cdot\text{s}$$

mutual inductance  $M$  depends on coils' geometry and magnetic properties of the material

# Transformers

- =two coils that share magnetic flux



$$\mathcal{E}_1 = N_1 \left| \frac{\Delta\Phi}{\Delta t} \right| \quad \mathcal{E}_2 = N_2 \left| \frac{\Delta\Phi}{\Delta t} \right|$$

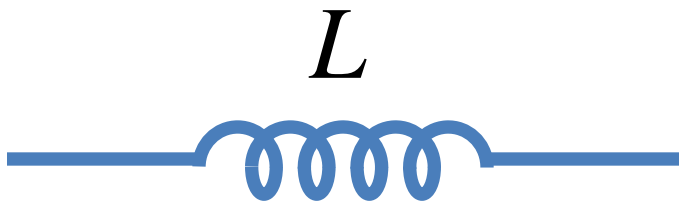
$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

# Self-Inductance

- =mutual inductance applied to the main coil itself

$$\mathcal{E} = L \left| \frac{\Delta i}{\Delta t} \right|$$

Inductor:



Capacitors vs inductors:



DC:  $i=0$  ( $R=\infty$ )


DC:  $v=0$  ( $R=0$ )

$$i = C \frac{\Delta v}{\Delta t}$$

$$v = L \frac{\Delta i}{\Delta t}$$

# Magnetic Field Energy

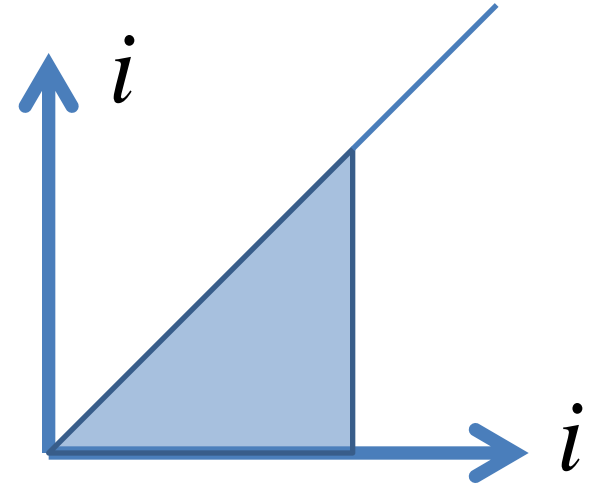
Power supplied during time  $\Delta t$ :


$$v = L \frac{\Delta i}{\Delta t}$$

$$P = vi = Li \frac{\Delta i}{\Delta t}$$

$$\Delta U = P\Delta t = Li\Delta i$$

$$U = \frac{1}{2} LI^2$$



# Capacitors vs Inductors



Energy

$$U = \frac{CV^2}{2}$$

$$U = \frac{LI^2}{2}$$

Energy density

$$u = \epsilon_0 \frac{E^2}{2}$$

$$u = \frac{B^2}{2\mu_0}$$

Mechanical analog

spring energy)

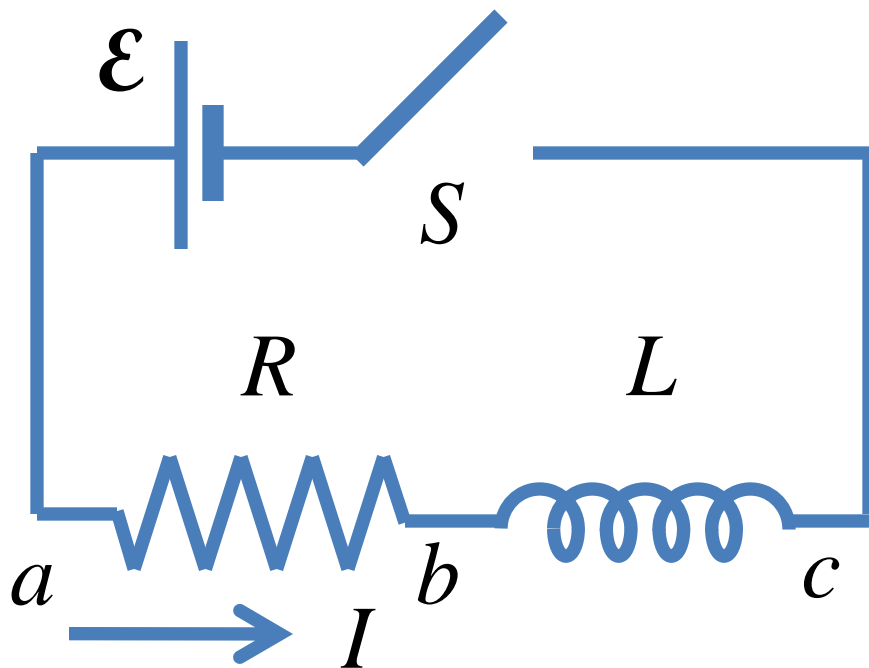
(potential mass energy)

(kinetic





# Resistance-Inductance Circuits



$$I = 0$$

$$V_{ab} = V_{bc} = 0$$

# Resistance-Inductance Circuits

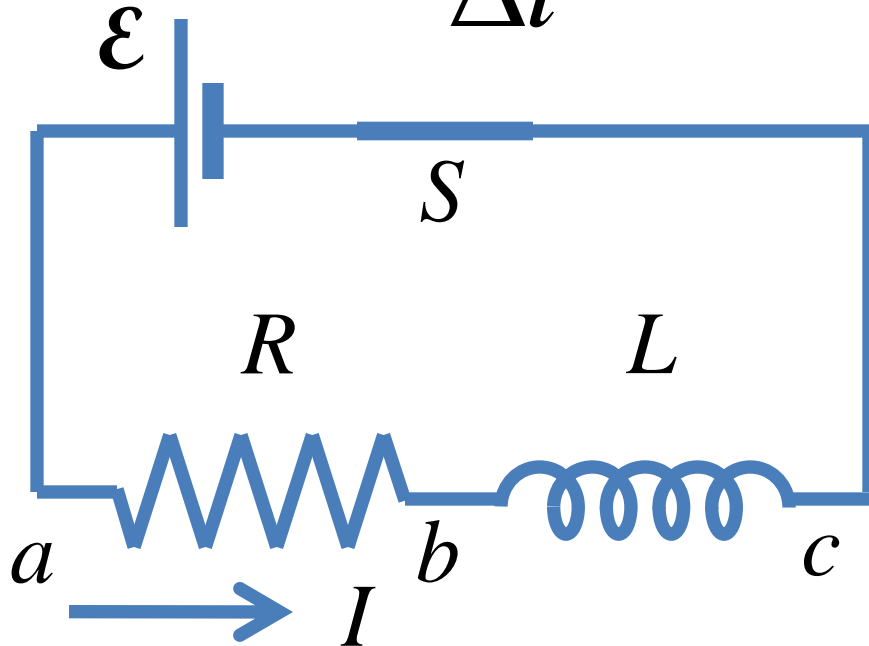
$$\mathcal{E} = iR + L \frac{\Delta i}{\Delta t}$$

Switch closed ( $t = 0$ )

$$i = 0$$

$$V_{ab} = 0$$

$$\left( \frac{\Delta i}{\Delta t} \right)_{initial} = \frac{\mathcal{E}}{L}$$



# Resistance-Inductance Circuits

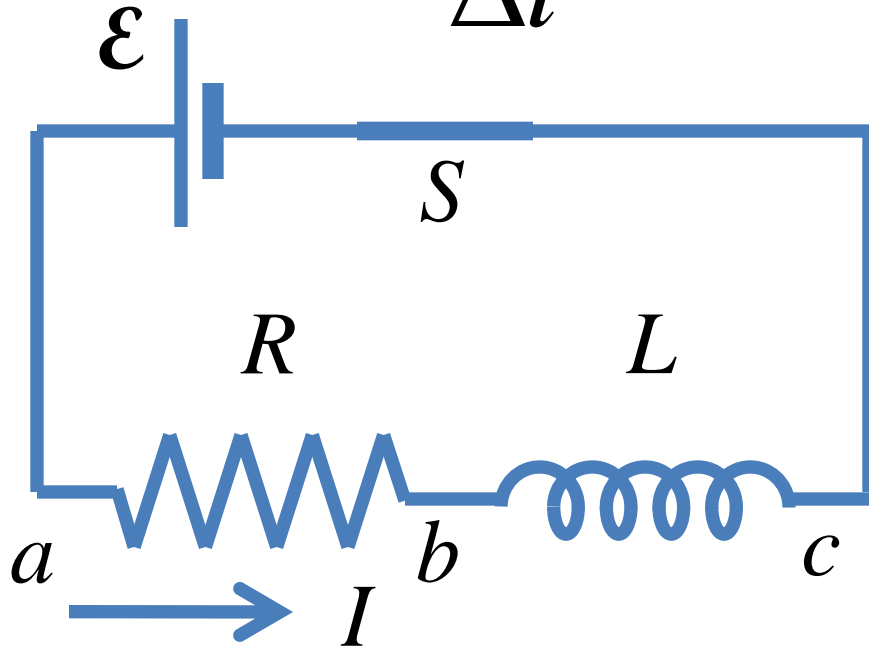
$$\mathcal{E} = iR + L \frac{\Delta i}{\Delta t}$$

Switch closed ( $t > 0$ )

$$i > 0$$

$$V_{ab} > 0$$

$$\frac{\Delta i}{\Delta t} = \frac{\mathcal{E}}{L} - \frac{R}{L} i$$

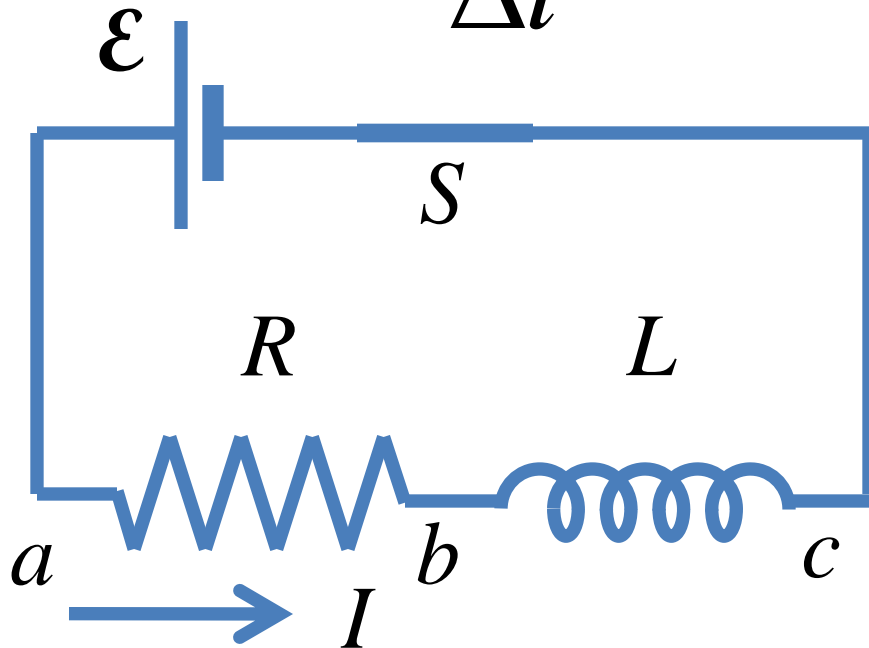


- current increases
- current change rate decreases

# Resistance-Inductance Circuits

$$\mathcal{E} = iR + L \frac{\Delta i}{\Delta t}$$

Switch closed ( $t = \infty$ )

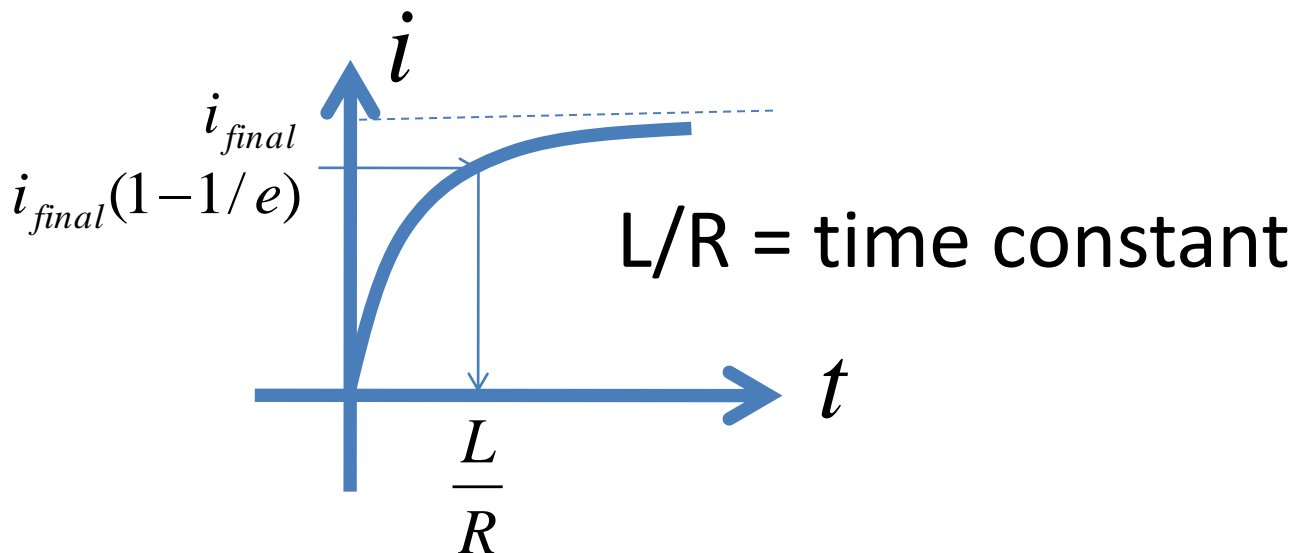


$$\left( \frac{\Delta i}{\Delta t} \right)_{final} = 0$$

$$i_{final} = \frac{\mathcal{E}}{R}$$

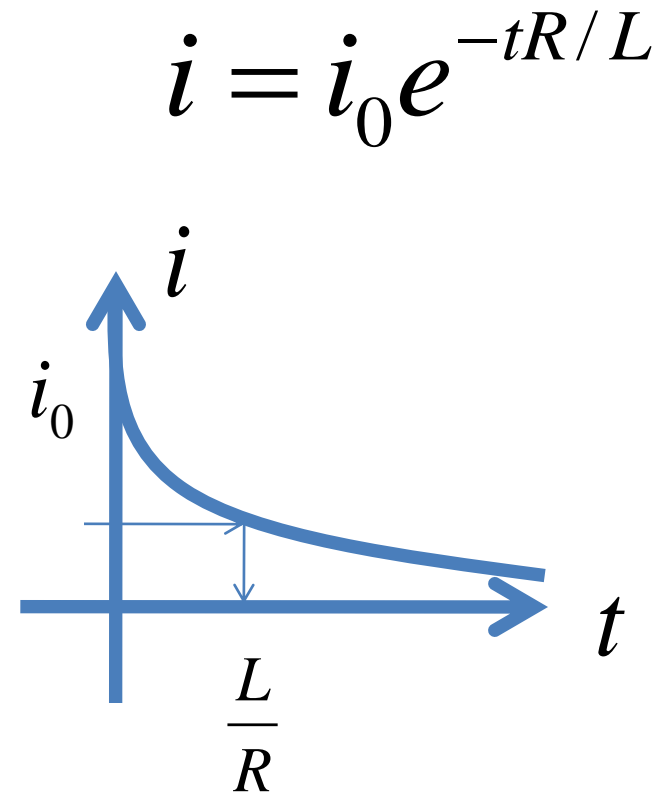
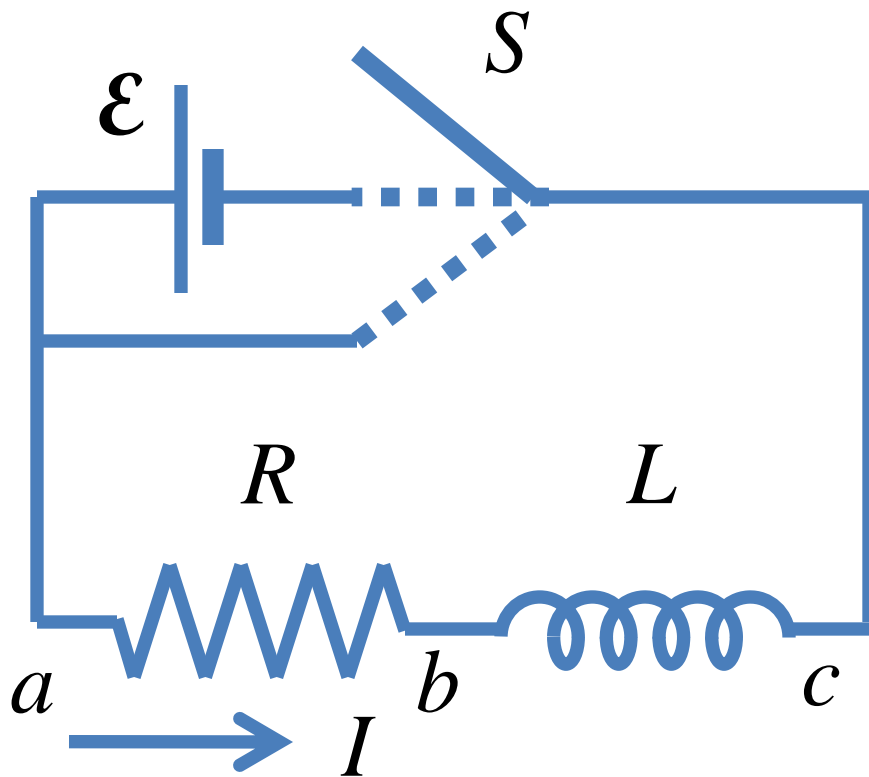
# Current vs Time

$$i = \frac{\mathcal{E}}{R} (1 - e^{-tR/L}) \quad \text{-- need calculus to derive it}$$



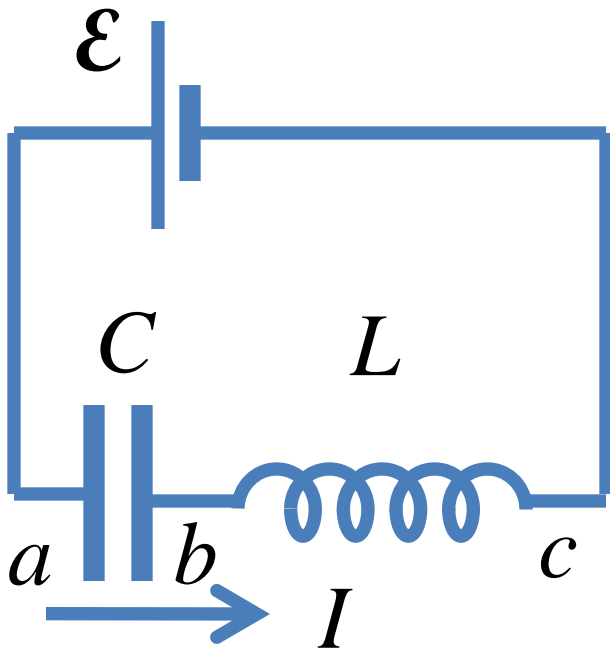
$$e = 2.71828\dots$$

# Current Decay in LR Circuit



# The L-C Circuit

$$E_{total} = \frac{CV^2}{2} + \frac{LI^2}{2}$$



$$\omega = \sqrt{\frac{1}{LC}}$$