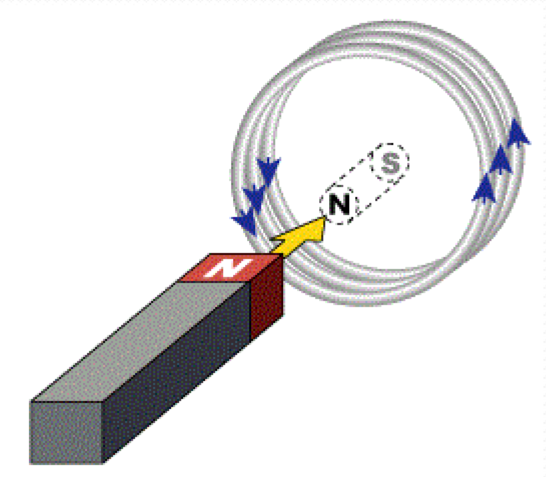
Chapter 21

Electromagnetic Induction

Induced EMF

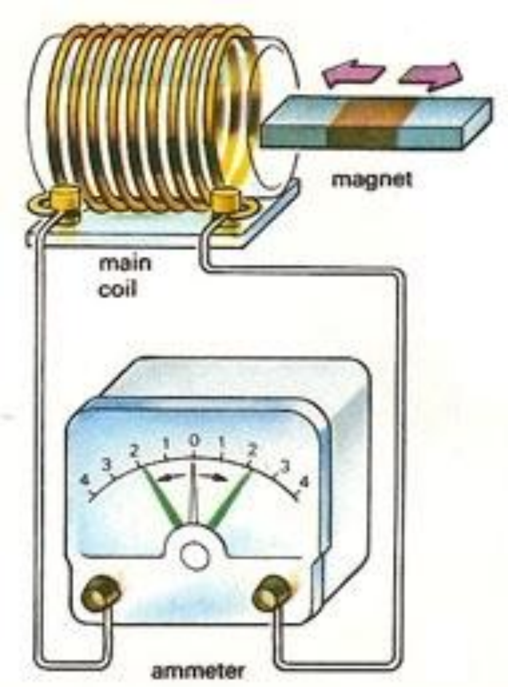
- We already know that moving charge (=current) causes magnetic field
- It also works the other way around: changing magnetic field (e.g. moving permanent magnet) causes current

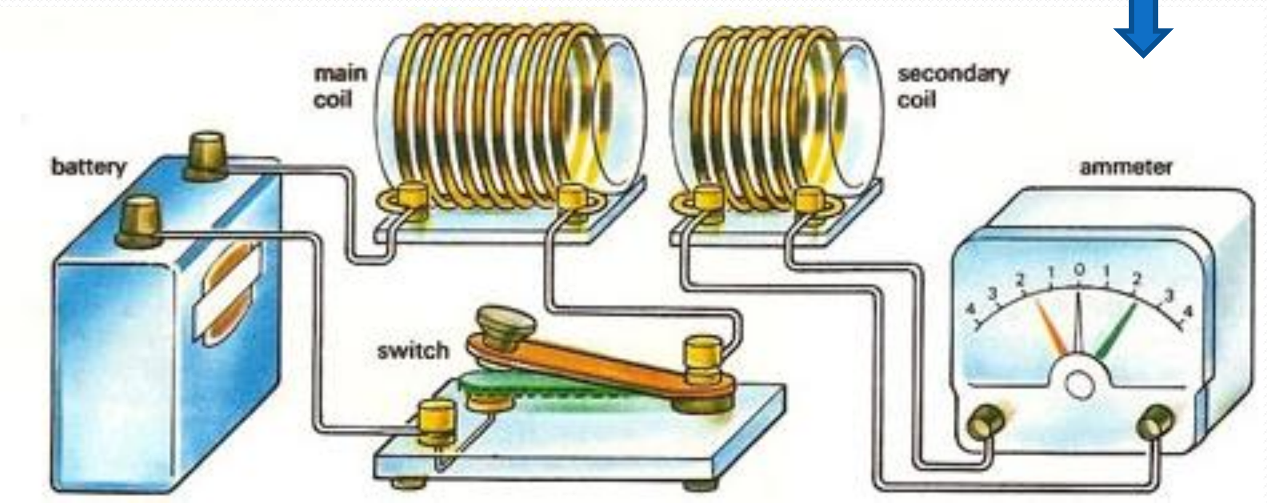


•it's called *induced current*
•since there is a current, there is electromotive force called *induced emf*

Induction Experiments

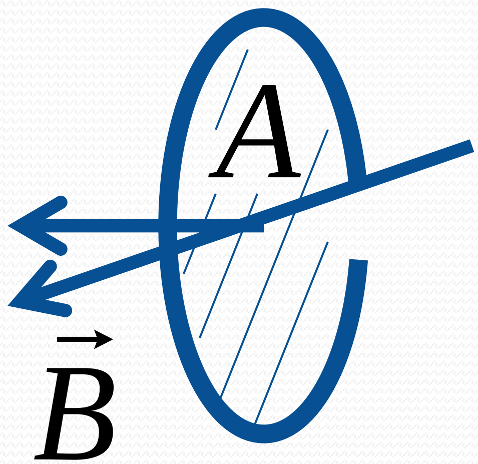
- Stationary permanent magnet does not induce current, the moving one does
- Stationary current in the first coil does not induce the current in the second coil, moving the coil in and out *or* switching the current on and off does





What matters is *change* of magnetic field

Magnetic Flux



$$\Delta\Phi = B\Delta A$$

if B is not perpendicular to surface then

$$\Delta\Phi = B\cos\varphi\,\Delta A$$

$$\Phi = \sum B\cos\varphi\,\Delta A$$

A is like a vector: you need to pick one of the two possible directions

if $B\cos\phi$ is the same everywhere then

$$\Phi = BA\cos\varphi$$

Faraday's Law

rate of flux change

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$

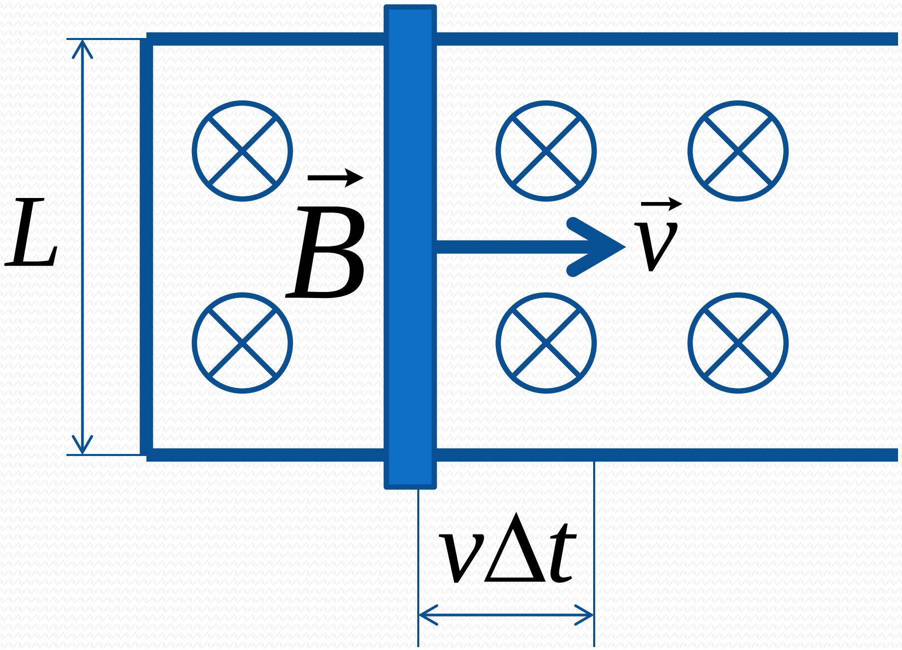
induced emf

for a coil of N turns, $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$

$[\Phi]$ = weber, 1 Wb = 1 T·m^2=1 V·s

notice the "–" sign in the formula

A Slide-Wire Generator



$$B = const$$

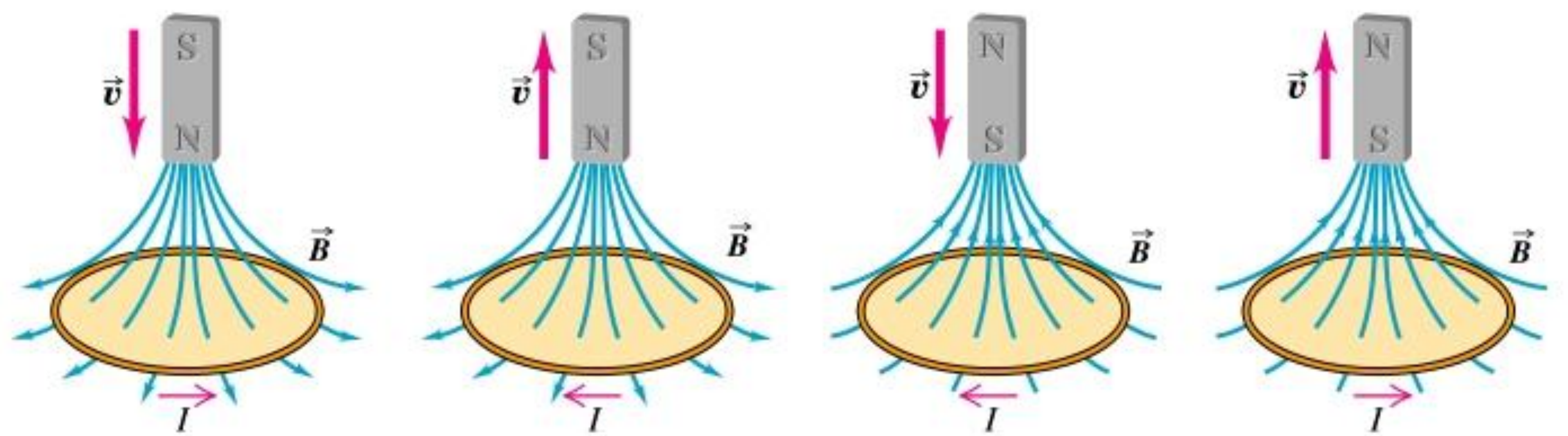
$$\Delta A = Lv\Delta t$$

$$\mathcal{E} = -\frac{B\Delta A}{\Delta t}$$

$$= -BLv$$

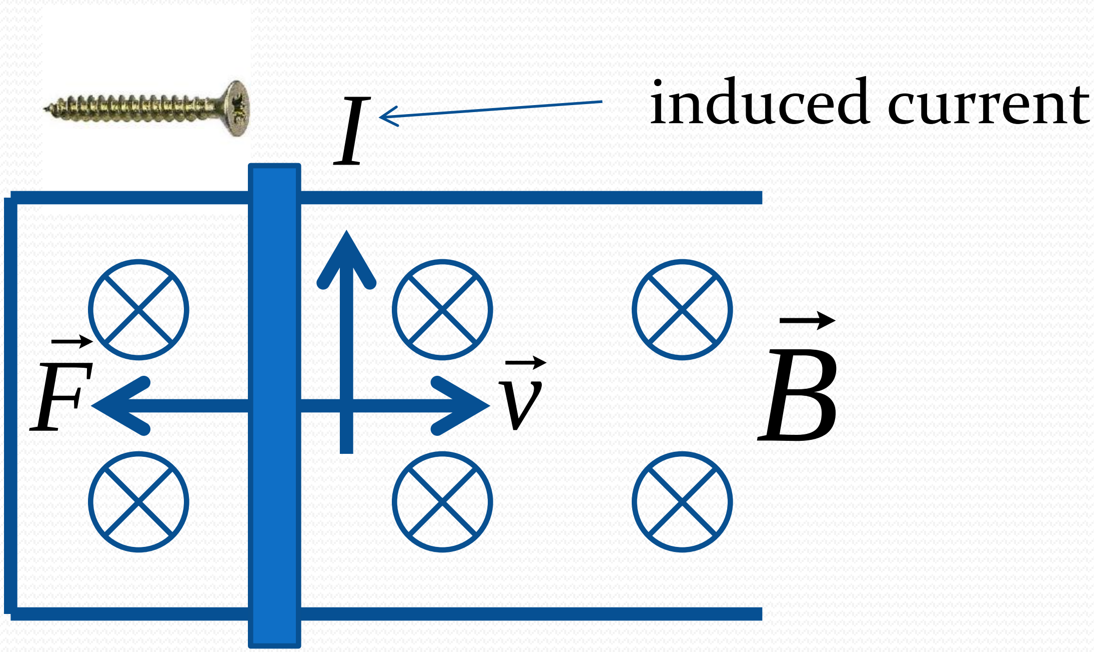
Lenz’s Law

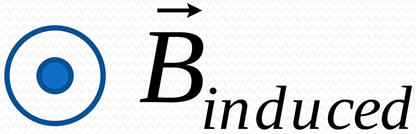
- The direction of induced current is such as to oppose the direction of the phenomenon causing it



Lenz’s law is a particular case of Le Chatelier's principle: in a stable equilibrium, any deviation would cause a force which drives the system back to the equilibrium (that’s why it’s stable!)

Lenz’s Law and a Slide-Wire rod



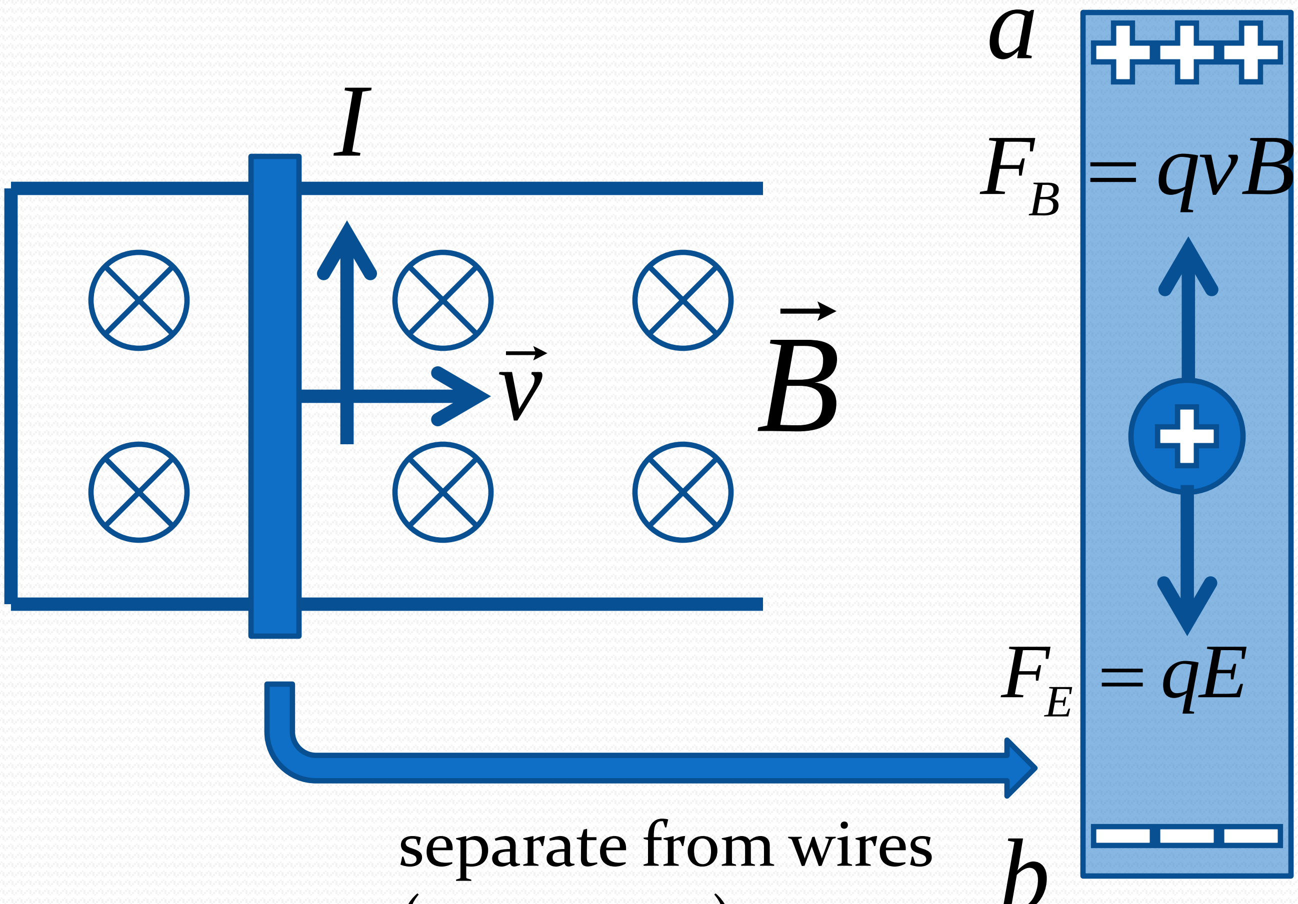


induced field is directed opposite to ***B***

direction of induced current is such that force ***F*** acting on it due to ***B*** tries to slow down the moving rod

otherwise we would get infinite acceleration without external force

Motional Electromotive Force



separate from wires
(no current)

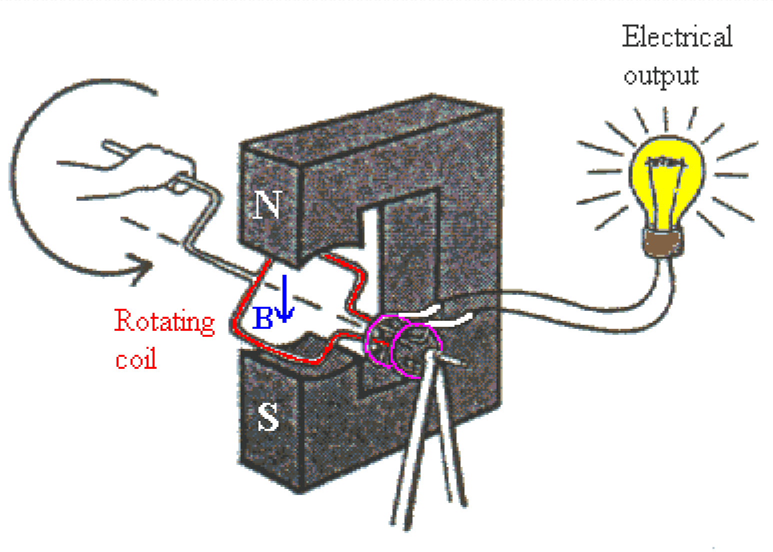
charges keep accumulating until electric force compensates the magnetic force

$$E = vB$$

$$V_{ab} = vBL$$

Generator

- =device which converts mechanical energy to electricity



$$\varphi = \omega t$$

$$\Phi = AB\cos\omega t$$

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t} = \omega AB\sin\omega t$$

need calculus to derive it

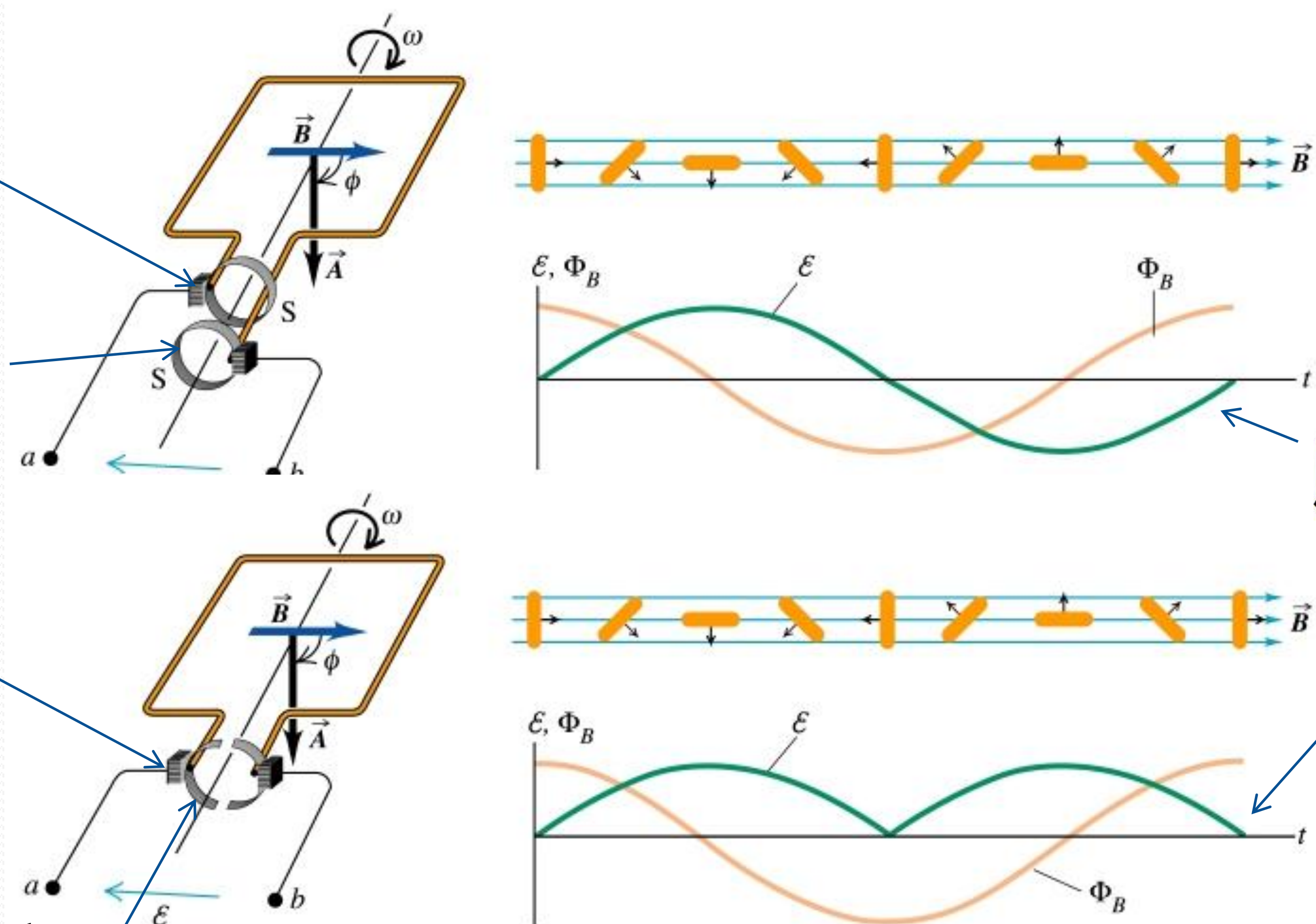
Alternating and Same Sign *emf*

brush

slip ring

brush

commutator



Mutual Inductance

- =coupling between two coils

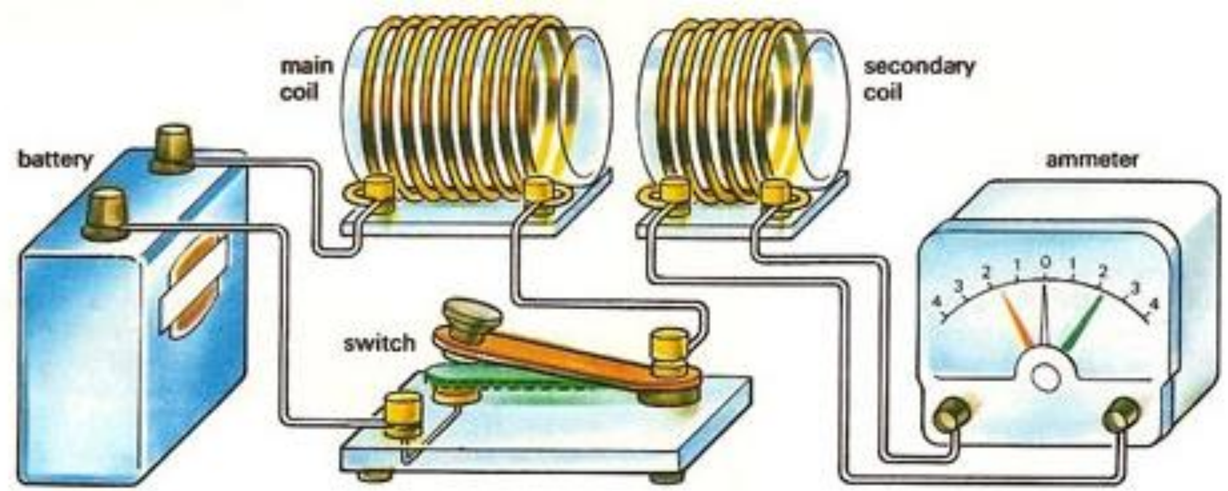
•when current through the main coil changes, the secondary coil gets induced emf
•the question is, how large is it?

$$\mathcal{E}_2 = N_2 \left| \frac{\Delta\Phi_2}{\Delta t} \right|$$

Φ_2 is proportional to the current in the main coil:

$$\Phi_2 = \frac{1}{N_2} M_{21} \, |\, i_1 \,|$$

for convenience



$$\mathcal{E}_2 = M_{21} \left| \frac{\Delta i_1}{\Delta t} \right|$$

Mutual Inductance

$$\mathcal{E}_2 = M_{21}\left|\frac{\Delta i_1}{\Delta t}\right| \qquad \mathcal{E}_1 = M_{12}\left|\frac{\Delta i_2}{\Delta t}\right|$$

it turns out that

$$M = M_{12} = M_{21}$$

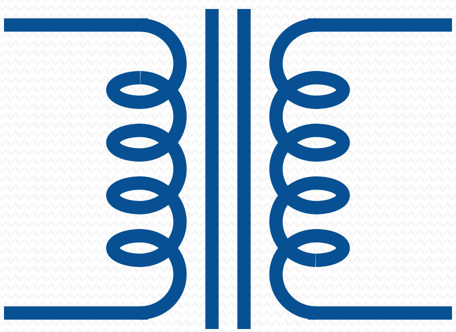
even for different coils

[M]= henry, 1 H = 1 Wb/A=1 V·s/A=1 Ω·s

mutual inductance *M* depends on coils' geometry and magnetic properties of the material

Transformers

- =two coils that share magnetic flux



$$\mathcal{E}_1 = N_1 \left| \frac{\Delta\Phi}{\Delta t} \right| \qquad \mathcal{E}_2 = N_2 \left| \frac{\Delta\Phi}{\Delta t} \right|$$

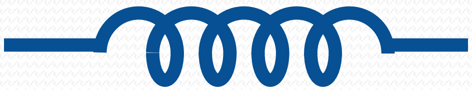
$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

Self-Inductance

- =mutual inductance applied to the main coil itself

$$\mathcal{E} = L\left|\frac{\Delta i}{\Delta t}\right|$$

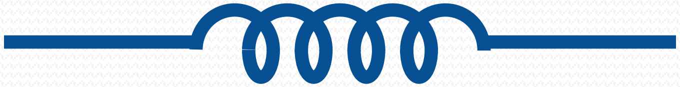
Capacitors vs inductors:



DC: i=0 (R=∞)   DC: v=0 (R=0)

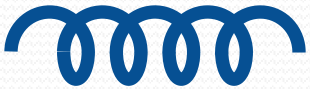
Inductor:

L



$$i = C\frac{\Delta v}{\Delta t} \qquad v = L\frac{\Delta i}{\Delta t}$$

Magnetic Field Energy

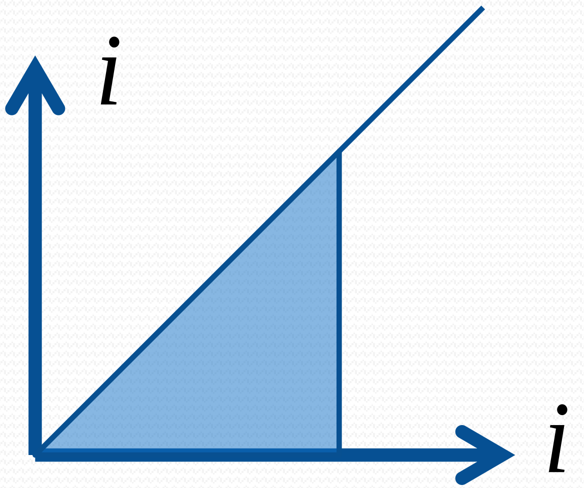


Power supplied during time Δt:

$$v = L\frac{\Delta i}{\Delta t}$$

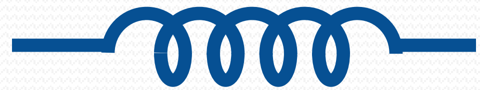
$$P = vi = Li\frac{\Delta i}{\Delta t}$$

$$\Delta U = P\Delta t = Li\Delta i$$



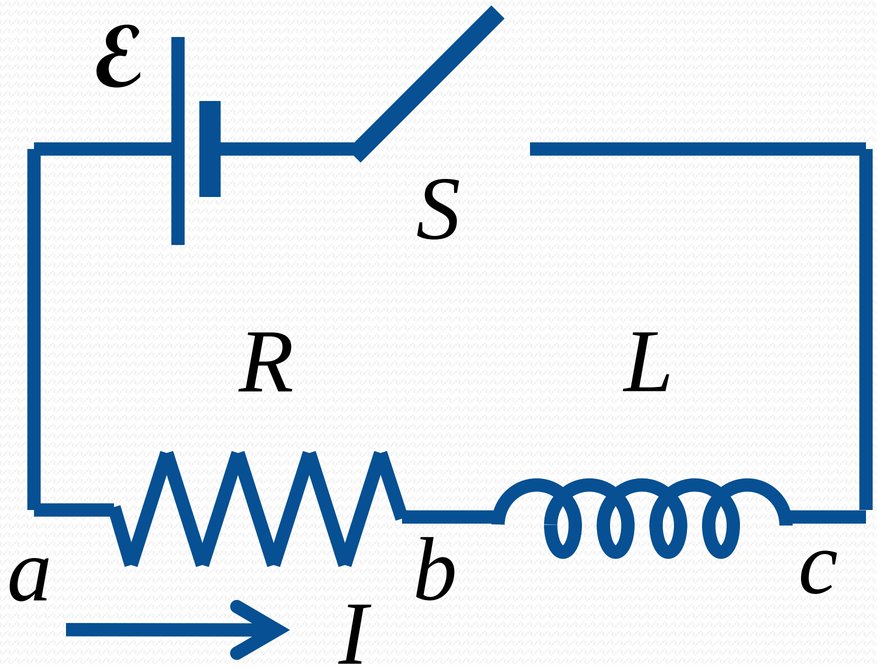
$$U = \frac{1}{2}LI^2$$

Capacitors vs Inductors

		
Energy	$U = \dfrac{CV^2}{2}$	$U = \dfrac{LI^2}{2}$
Energy density	$u = \varepsilon_0 \dfrac{E^2}{2}$	$u = \dfrac{B^2}{2\mu_0}$
Mechanical analog	spring (potential energy)	mass (kinetic energy)

☺

Resistance-Inductance Circuits

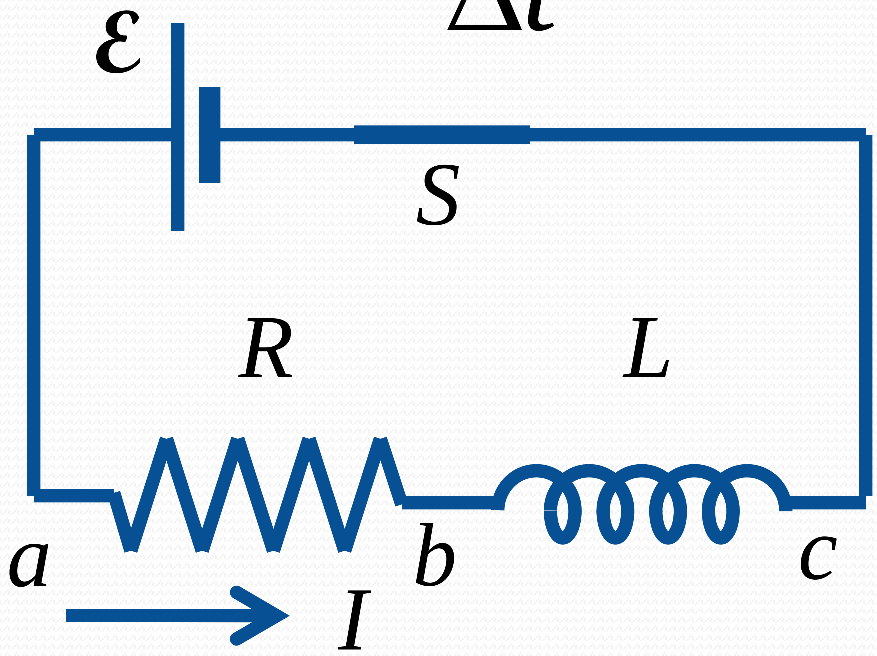


$$I = 0$$

$$V_{ab} = V_{bc} = 0$$

Resistance-Inductance Circuits

$$\mathcal{E} = iR + L\frac{\Delta i}{\Delta t}$$



Switch closed $(t = 0)$

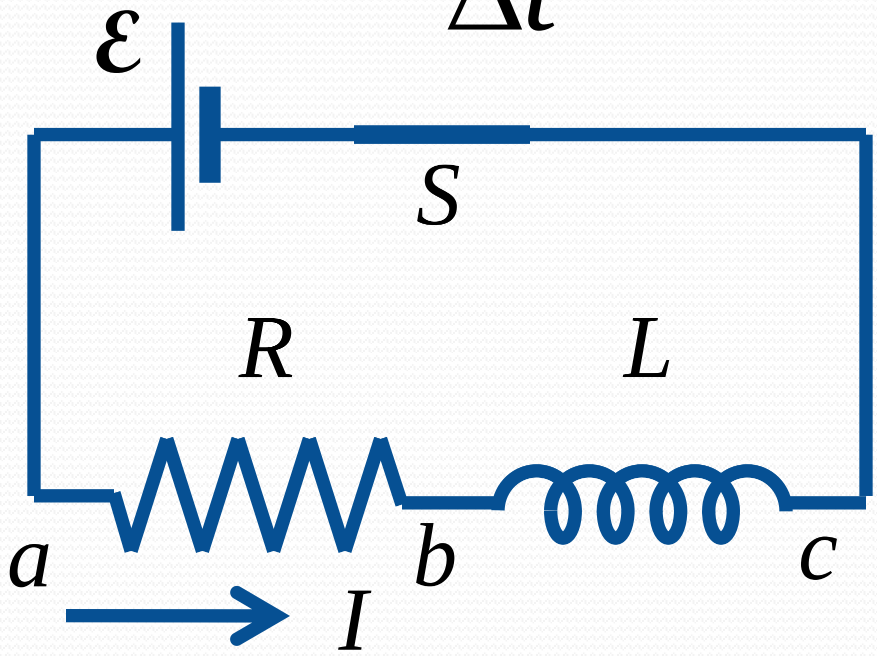
$$i = 0$$

$$V_{ab} = 0$$

$$\left(\frac{\Delta i}{\Delta t}\right)_{initial} = \frac{\mathcal{E}}{L}$$

Resistance-Inductance Circuits

$$\mathcal{E} = iR + L\frac{\Delta i}{\Delta t}$$



Switch closed $(t > 0)$

$$i > 0$$

$$V_{ab} > 0$$

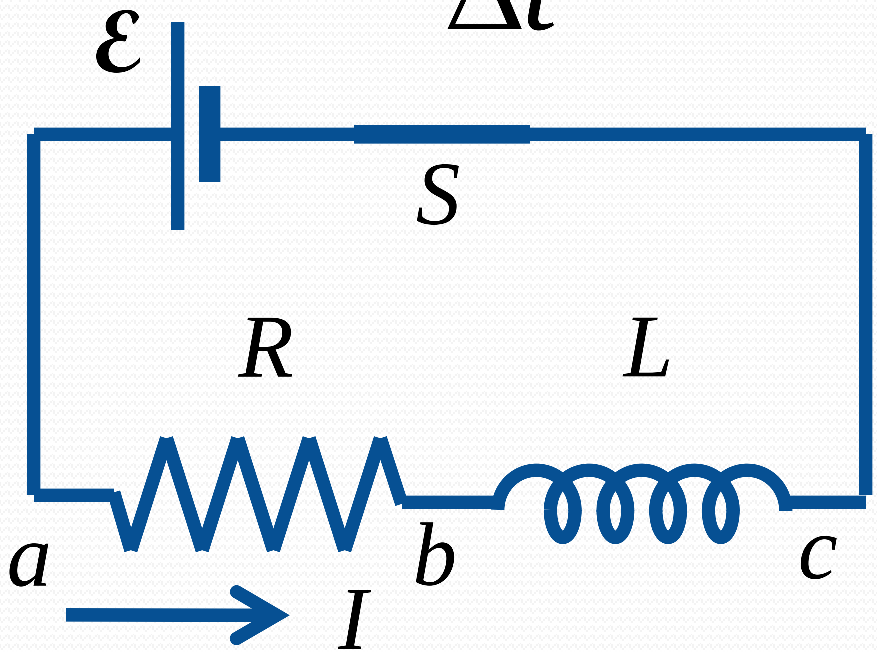
$$\frac{\Delta i}{\Delta t} = \frac{\mathcal{E}}{L} - \frac{R}{L}i$$

- current increases
- current change rate decreases

Resistance-Inductance Circuits

$$\mathcal{E} = iR + L\frac{\Delta i}{\Delta t}$$

Switch closed $(t = \infty)$

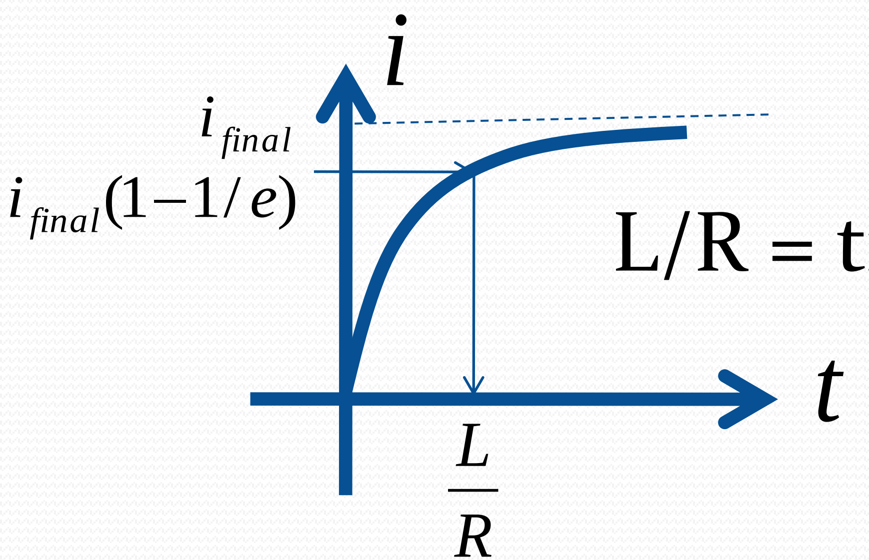


$$\left(\frac{\Delta i}{\Delta t}\right)_{final} = 0$$

$$i_{final} = \frac{\mathcal{E}}{R}$$

Current vs Time

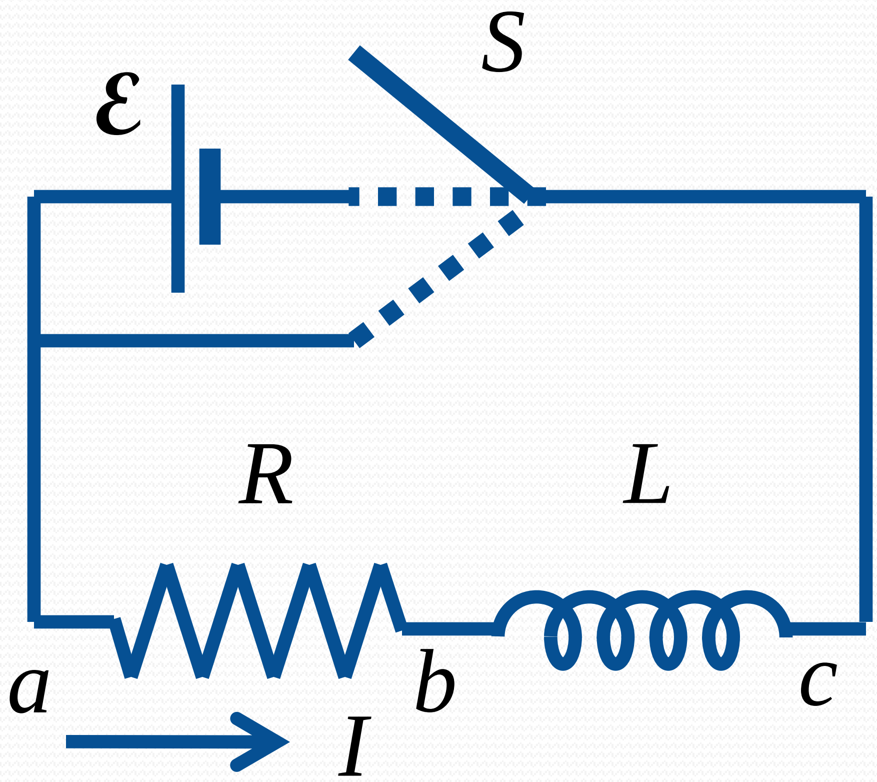
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$ – need calculus to derive it



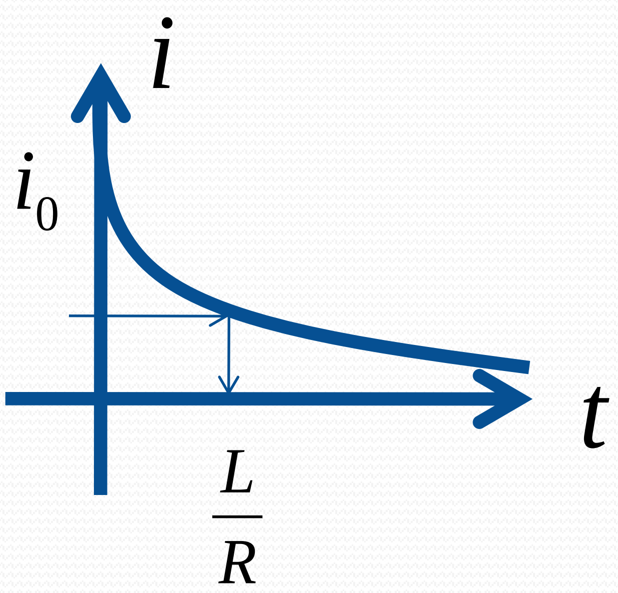
$e = 2.71828\ldots$

L/R = time constant

Current Decay in LR Circuit



$$i = i_0 e^{-tR/L}$$



The L-C Circuit

$$E_{total} = \frac{CV^2}{2} + \frac{LI^2}{2}$$

$$\omega = \sqrt{\frac{1}{LC}}$$