Chapter 21
Electromagnetic Induction
Induced EMF

- We already know that moving charge (=current) causes magnetic field
- It also works the other way around: changing magnetic field (e.g. moving permanent magnet) causes current
  - it’s called *induced current*
  - since there is a current, there is electromotive force called *induced emf*
Induction Experiments

- Stationary permanent magnet does not induce current, the moving one does.
- Stationary current in the first coil does not induce the current in the second coil, moving the coil in and out or switching the current on and off does.

What matters is change of magnetic field.
Magnetic Flux

\[ \Delta \Phi = B \Delta A \]

if \( B \) is not perpendicular to surface then

\[ \Delta \Phi = B \cos \phi \Delta A \]

\[ \Phi = \sum B \cos \phi \Delta A \]

if \( B \cos \phi \) is the same everywhere then

\[ \Phi = BA \cos \phi \]

A is like a vector: you need to pick one of the two possible directions
Faraday’s Law

\[ \mathcal{E} = -\frac{\Delta \Phi}{\Delta t} \]

for a coil of \( N \) turns, \( \mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} \)

induced emf

rate of flux change

\([\Phi] = \text{weber}, 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2 = 1 \text{ V} \cdot \text{s}\)

notice the “−” sign in the formula
A Slide-Wire Generator

\[ B = \text{const} \]
\[ \Delta A = L v \Delta t \]
\[ \mathcal{E} = - \frac{B \Delta A}{\Delta t} \]
\[ = -BLv \]
Lenz’s Law

- The direction of induced current is such as to oppose the direction of the phenomenon causing it.

Lenz’s law is a particular case of Le Chatelier's principle: in a stable equilibrium, any deviation would cause a force which drives the system back to the equilibrium (that’s why it’s stable!)
Lenz's Law and a Slide-Wire rod

Direction of induced current is such that force $F$ acting on it due to $B$ tries to slow down the moving rod; otherwise we would get infinite acceleration without external force.
Motional Electromotive Force

charges keep accumulating until electric force compensates the magnetic force

\[ F_B = qvB \]

\[ F_E = qE \]

\[ E = \nu B \]

\[ V_{ab} = \nu BL \]
Generator

- device which converts mechanical energy to electricity

\[ \varphi = \omega t \]

\[ \Phi = AB \cos \omega t \]

\[ \mathcal{E} = -\frac{\Delta \Phi}{\Delta t} = \omega AB \sin \omega t \]

need calculus to derive it
Alternating and Same Sign $emf$
Mutual Inductance

- coupling between two coils

- when current through the main coil changes, the secondary coil gets induced emf

- the question is, how large is it?

\[ \mathcal{E}_2 = N_2 \left| \frac{\Delta \Phi_2}{\Delta t} \right| \]

\( \Phi_2 \) is proportional to the current in the main coil:

\[ \Phi_2 = \frac{1}{N_2} M_{21} |i_1| \]

for convenience

\[ \mathcal{E}_2 = M_{21} \left| \frac{\Delta i_1}{\Delta t} \right| \]
Mutual Inductance

\[ \mathcal{E}_2 = M_{21} \left| \frac{\Delta i_1}{\Delta t} \right| \]
\[ \mathcal{E}_1 = M_{12} \left| \frac{\Delta i_2}{\Delta t} \right| \]

it turns out that
\[ M = M_{12} = M_{21} \]
even for different coils

\[ [M] = \text{henry}, \ 1 \text{ H} = 1 \text{ Wb}/\text{A}=1 \text{ V} \cdot \text{s}/\text{A}=1 \Omega \cdot \text{s} \]

mutual inductance \( M \) depends on coils’ geometry and magnetic properties of the material
Transformers

= two coils that share magnetic flux

\[ \mathcal{E}_1 = N_1 \left| \frac{\Delta \Phi}{\Delta t} \right| \quad \mathcal{E}_2 = N_2 \left| \frac{\Delta \Phi}{\Delta t} \right| \]

\[ \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \]
Self-Inductance

- mutual inductance applied to the main coil itself

\[ \mathcal{E} = L \left| \frac{\Delta i}{\Delta t} \right| \]

Inductor:

\[ i = C \frac{\Delta v}{\Delta t} \quad v = L \frac{\Delta i}{\Delta t} \]

Capacitors vs inductors:

DC: \( i = 0 \) (\( R = \infty \))  \quad \text{DC: } v = 0 \ (R = 0)
Magnetic Field Energy

Power supplied during time $\Delta t$:

$$P = vi = Li \frac{\Delta i}{\Delta t}$$

$$\Delta U = P \Delta t = Li \Delta i$$

$$U = \frac{1}{2} LI^2$$
Capacitors vs Inductors

Energy

\[ U = \frac{CV^2}{2} \]

\[ U = \frac{LI^2}{2} \]

Energy density

\[ u = \varepsilon_0 \frac{E^2}{2} \]

\[ u = \frac{B^2}{2\mu_0} \]

Mechanical analog

spring (potential energy)

mass (kinetic energy)

Electromagnetic Induction
Resistence-Inductance Circuits

\[ I = 0 \]

\[ V_{ab} = V_{bc} = 0 \]
Resistance-Inductance Circuits

\[ \mathcal{E} = iR + L \frac{\Delta i}{\Delta t} \]

Switch closed \((t = 0)\)

\[ i = 0 \]

\[ V_{ab} = 0 \]

\[ \left( \frac{\Delta i}{\Delta t} \right)_{initial} = \frac{\mathcal{E}}{L} \]
Resistance-Inductance Circuits

\[ \mathcal{E} = iR + L \frac{\Delta i}{\Delta t} \]

Switch closed \((t > 0)\)

\[ i > 0 \]

\[ V_{ab} > 0 \]

\[ \frac{\Delta i}{\Delta t} = \frac{\mathcal{E}}{L} - \frac{R}{L} i \]

- current increases
- current change rate decreases
Resistance-Inductance Circuits

\[ \mathcal{E} = iR + L \frac{\Delta i}{\Delta t} \]

Switch closed \((t = \infty)\)

\[ \left( \frac{\Delta i}{\Delta t} \right)_{\text{final}} = 0 \]

\[ i_{\text{final}} = \frac{\mathcal{E}}{R} \]
Current vs Time

\[ i = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{tR}{L}}\right) \] - need calculus to derive it

\[ e = 2.71828 \ldots \]

L/R = time constant
Current Decay in LR Circuit

\[ i = i_0 e^{-\frac{tR}{L}} \]
The L-C Circuit

\[ E_{total} = \frac{CV^2}{2} + \frac{LI^2}{2} \]

\[ \omega = \sqrt{\frac{1}{LC}} \]