Chapter 22

Electromagnetic Waves
Maxwell’s Hypothesis

• What was known: Faraday’s Law
  – changing magnetic field $B$ induces electric field $E$

$$\mathcal{E} = -\frac{\Delta \Phi}{\Delta t}$$

• What was proposed by Maxwell:
  – changing electric field $E$ induces magnetic field $B$
Displacement Currents

Due to real conduction current $I$

Due to imaginary displacement current $I_D$

$B = \frac{\mu_0 I}{2\pi r}$

due to real conduction current $I$

due to imaginary displacement current $I_D$

$\Delta E = \varepsilon_0 \frac{\Delta E}{\Delta t} A$

Rate of change of electric flux $\Phi_E = EA$
Maxwell’s Equations

• By combining known facts (changing $B$ produces $E$) and his hypothesis (changing $E$ produces $B$), Maxwell was able to write down a nice system of equations relating $E$ and $B$
  – I won’t show them to you 😊
• The solution of these equations is $B$ and $E$ as functions of $x, y, z, t$
  – It turns out the solution looks like waves what is that?
Electromagnetic Wave

• Simplest solution of Maxwell’s equations without any sources (charges/currents):

\[
\begin{align*}
E_x &= 0, \quad E_y = E_{\text{max}} \sin(\omega t - kx), \quad E_z = 0 \\
B_x &= 0, \quad B_y = 0, \quad B_z = B_{\text{max}} \sin(\omega t - kx)
\end{align*}
\]
Electromagnetic Wave

\[ E = E_{\text{max}} \sin(\omega t - kx) \]

- **Fix** \( t=0 \)
- Wave number \( k = \frac{2\pi}{\lambda} \)
- Wavelength \( \lambda \)
Electromagnetic Wave

\[ E = E_{\text{max}} \sin(\omega t - kx) \]

\[ \omega = \frac{2\pi}{T} \]
Electromagnetic Wave

\[ E = E_{\text{max}} \sin(\omega t - kx) \]

It's moving! It's speed is

\[ c = \frac{\omega}{k} \]

Frequency

\[ f = \frac{\omega}{2\pi} \]

\[ c = \lambda f \]
Electromagnetic Wave

\[ E_y = E_{\text{max}} \sin(\omega t - kx) \]

\[ B_z = B_{\text{max}} \sin(\omega t - kx) \]
Relation between $E$ and $B$

$$E_{\text{max}} = cB_{\text{max}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$c = \frac{1}{\sqrt{[8.85 \times 10^{-12} \text{C}^2 / (\text{N} \cdot \text{m}^2)] [4\pi \times 10^{-7} \text{N/A}^2]}} = 3 \times 10^8 \text{ m/s}$$

It’s speed of light!

$\varepsilon_0$ and $\mu_0$ are obtained from measurements of electric and magnetic forces. The fact that the speed of light (an experimental quantity) $c=1/\sqrt{\varepsilon_0 \mu_0}$ tells us that the light has electromagnetic nature.
Energy Density in EM Waves

\[ u = u_E + u_B = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2 \mu_0} B^2 \]

\[ E = Bc \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]

Energy densities associated with \( E \) and \( B \) are the same

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} = \frac{1}{2 \mu_0} B^2 = u_B \]
EM Energy Flow

- **S**: energy flow per unit time per unit area

\[
S = \frac{1}{A} \frac{\Delta U}{\Delta t} \quad \Delta U = u\Delta V = u(Ac\Delta t) \quad S = uc
\]

\[
u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}
\]

\[
S = \varepsilon_0 E^2 c = \frac{B^2}{\mu_0} c = \frac{EB}{\mu_0}
\]
Properties of EM Waves

- Electromagnetic waves carry energy

\[ I = \langle S \rangle = \langle u \rangle c = \frac{1}{2} \frac{E_{\text{max}} B_{\text{max}}}{\mu_0} \]

- Electromagnetic waves also carry momentum and exhibit pressure

\[ \frac{\langle p \rangle}{V} = \frac{I}{c^2} \quad \text{radiation pressure:} \quad P = \frac{I}{c} \]

sunlight pressure is ~$10^{-10}$ atm
Are EM Waves Real?

- Yes!

Hertz, in his experiments (1887),

- demonstrated transmission of EM waves;

- measured speed of EM waves and showed it to be the same as the speed of light*;

- studied wave properties of EM radiation (reflection, standing waves).

Few years later, radio was invented

* already measured by that time using gears or rotating mirrors.