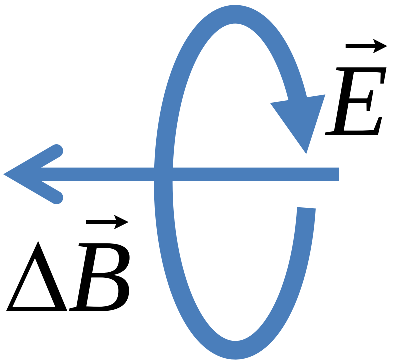
Chapter 22

Electromagnetic Waves

Maxwell's Hypothesis

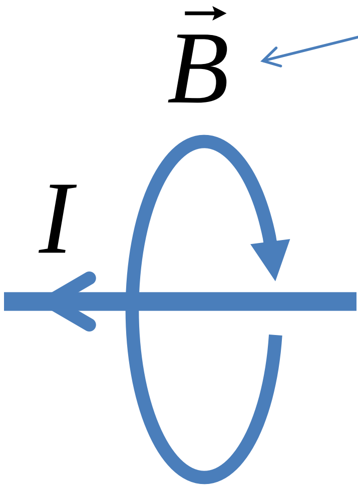
- What was known: Faraday's Law
  - changing magnetic field ***B*** induces electric field ***E***

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$



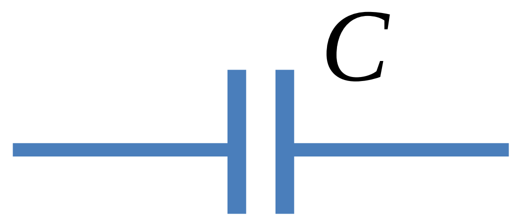
- What was proposed by Maxwell:
  - changing electric field ***E*** induces magnetic field ***B***

Displacement Currents

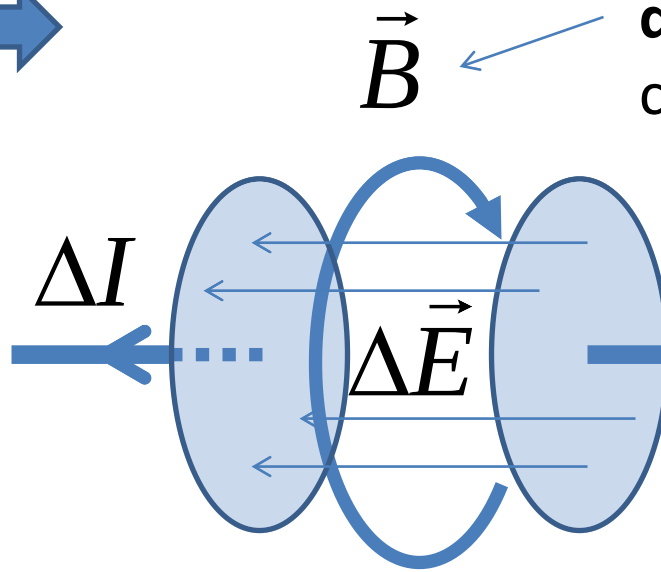


due to real **conduction** current ***I***

$$B = \frac{\mu_0 I}{2\pi r}$$



due to imaginary **displacement** current I_D



$$I_D = \varepsilon_0 \frac{\Delta E}{\Delta t} A$$

rate of change of electric flux $\Phi_E = EA$

Maxwell's Equations

- By combining known facts (changing ***B*** produces ***E***) and his hypothesis (changing ***E*** produces ***B***), Maxwell was able to write down a nice system of equations relating ***E*** and ***B***
  – I won't show them to you ☺
- The solution of these equations is ***B*** and ***E*** as functions of *x*,*y*,*z*,*t*
  – It turns out the solution looks like waves

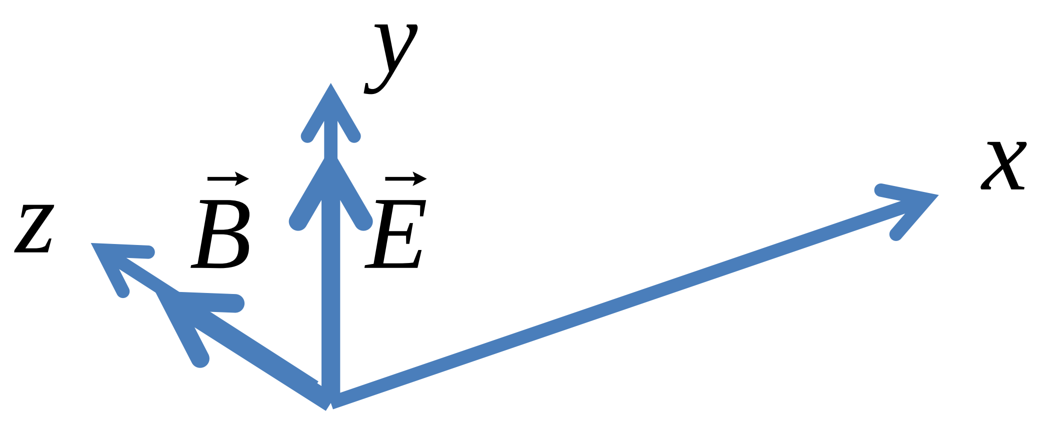
what is that?

Electromagnetic Wave

- Simplest solution of Maxwell's equations without any sources (charges/currents):

$$E_x = 0, \quad E_y = E_{\max} \sin(\omega t - kx), \quad E_z = 0$$

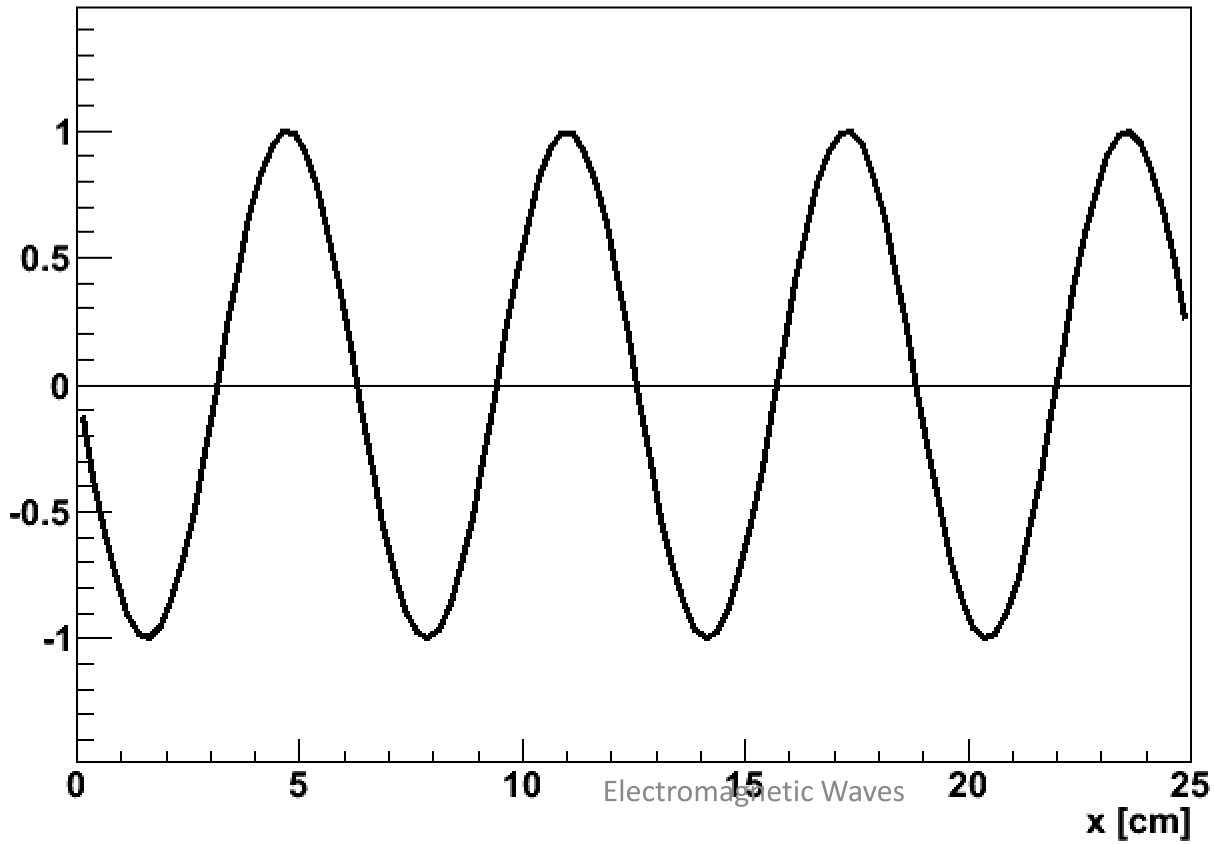
$$B_x = 0, \quad B_y = 0, \quad B_z = B_{\max} \sin(\omega t - kx)$$



Electromagnetic Wave

fix ***t=0***

$$E = E_{\max} \sin(\omega t - kx)$$



wave number

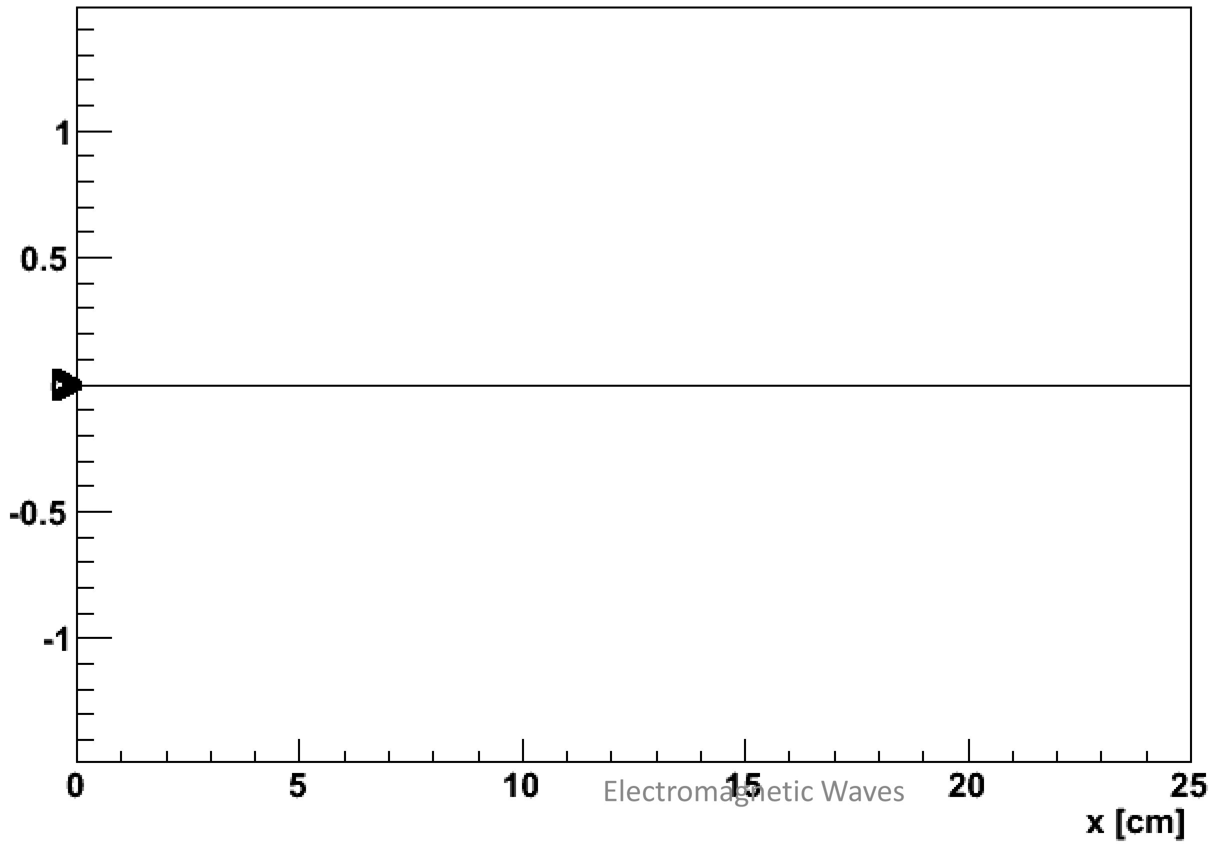
$$k = \frac{2\pi}{\lambda}$$

wavelength

Electromagnetic Wave

fix ***x=0***

$$E = E_{\max}\sin(\omega t - kx)$$



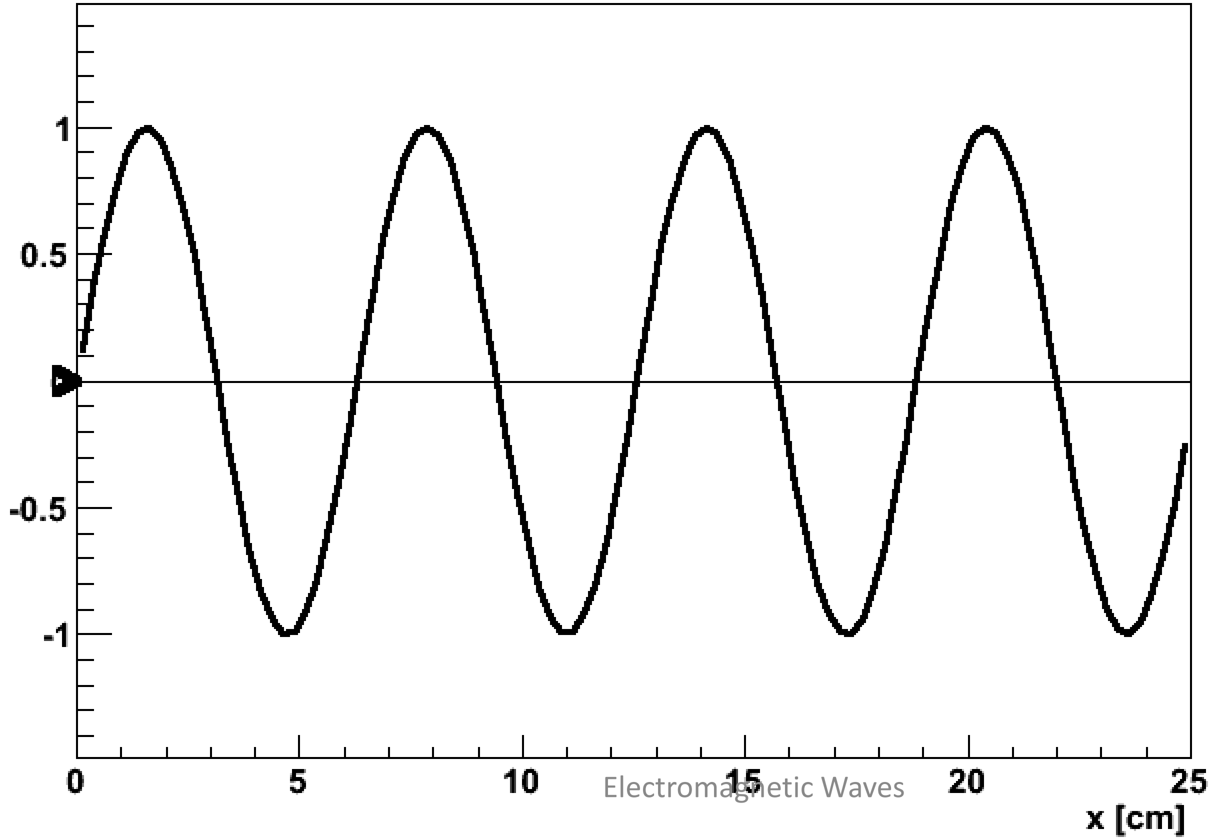
angular frequency

$$\omega = \frac{2\pi}{T}$$

period

Electromagnetic Wave

$$E = E_{\max} \sin(\omega t - kx)$$



it's moving!
it's speed is

$$c = \frac{\omega}{k}$$

frequency

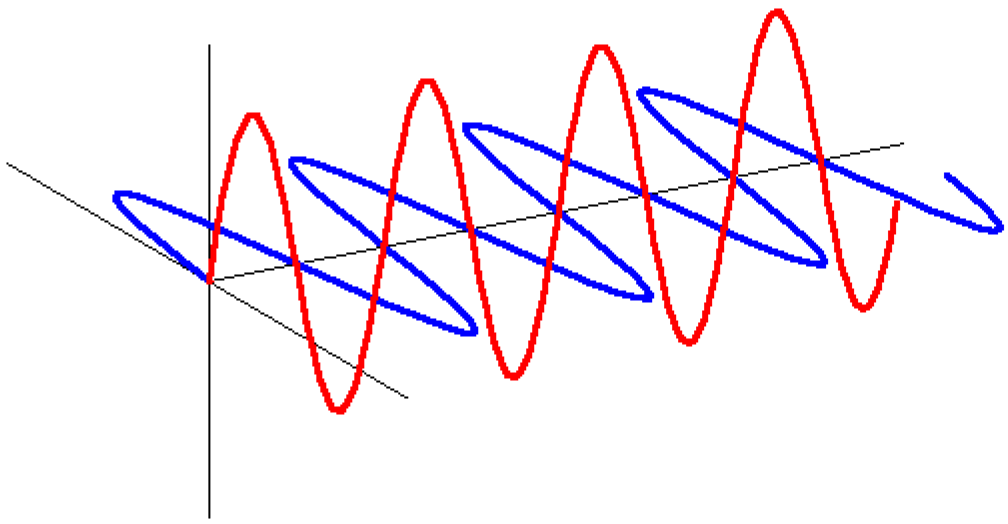
$$f = \frac{\omega}{2\pi}$$

$$c = \lambda f$$

Electromagnetic Wave

$$E_y = E_{\max} \sin(\omega t - kx)$$

$$B_z = B_{\max} \sin(\omega t - kx)$$



Relation between *E* and *B*

$$E_{\max} = cB_{\max} \qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$c = \frac{1}{\sqrt{[8.85\times10^{-12}\,\mathrm{C}^2/(\mathrm{N}\cdot\mathrm{m}^2)][4\pi\times10^{-7}\,\mathrm{N/A}^2]}} = 3\times10^{8}\,\mathrm{m/s}$$

it's speed of light!

ε_0 and μ_0 are obtained from measurements of electric and magnetic forces
The fact that the speed of light (an experimental quantity) $c=1/\sqrt{\varepsilon_0\mu_0}$ tells us that the light has electromagnetic nature

Energy Density in EM Waves

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$E = Bc \qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Energy densities associated with ***E*** and ***B*** are the same

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} = \frac{1}{2\mu_0} B^2 = u_B$$

EM Energy Flow

- ***S***: energy flow per unit time per unit area

$$S = \frac{1}{A}\frac{\Delta U}{\Delta t} \quad \Delta U = u\Delta V = u(Ac\Delta t) \quad S = uc$$

$$u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \qquad S = \varepsilon_0 E^2 c = \frac{B^2}{\mu_0}c = \frac{EB}{\mu_0}$$

Properties of EM Waves

- Electromagnetic waves carry energy

intensity
$$I = \langle S \rangle = \langle u \rangle c = \frac{1}{2}\frac{E_{max} B_{max}}{\mu_0}$$

- Electromagnetic waves also carry momentum and exhibit pressure

momentum density:
$$\frac{\langle p \rangle}{V} = \frac{I}{c^2}$$

radiation pressure:
$$P = \frac{I}{c}$$

sunlight pressure is ~10^{-10} atm

Are EM Waves Real?

- Yes!

Hertz, in his experiments (1887),

➢demonstrated transmission of EM waves;

➢measured speed of EM waves and showed it to be the same as the speed of light*;

➢studied wave properties of EM radiation (reflection, standing waves).

Few years later, radio was invented

* already measured by that time using gears or rotating mirrors