Chapter 23

Geometric Optics
Light

• What is light? Waves or particles?
  
  both

• Geometric optics: light travels in straight-line paths called rays
  
  – This is true if typical distances are much larger than the wavelength
What it is about

• Phenomena addressed by geometric optics:
  – Propagation: light goes in straight lines
  – Reflection: angle of incidence = angle of reflection
  – Refraction: Snell’s law

• Phenomena not addressed by geometric optics:
  – Polarization
  – Diffraction
  – Interference
  – Photoelectric effect

Electromagnetic theory

Quantum mechanics
Reflection and Refraction

The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane.
Specular and Diffusive Reflection

*specular* reflection: smooth interface, definite reflection angle

*diffusive* reflection: rough interface, scattered reflection

we’ll discuss this one
**Image**

**Object point:** where the rays actually come from

**Image point:** where the rays appear to come from

$s$: object distance

$s'$: image distance

Here, outgoing rays do not actually come from $P'$ → image is **virtual**

If they did, the image would be **real**
Sign Rules for Distances

Object distance:
when the object is on the same side as the incoming light, $s>0$, otherwise, $s<0$

Image distance:
when the image is on the same side as the outgoing light, $s'>0$, otherwise, $s'<0$

object point $P$

image point $P'$
mirror

$s > 0$

$s' < 0$
Sign Rules for Distances

Object distance:
- when the object is on the same side as the incoming light, $s > o$, otherwise, $s < o$

Image distance:
- when the image is on the same side as the outgoing light, $s' > o$, otherwise, $s' < o$
Lateral Magnification

need at least two points \((P,Q)\) to figure it out

mirror:
\[ s = -s' \]
\[ m = 1 \]

\[ m = \frac{y'}{y} \]
Inverted and Reversed Images

• Image can be **erect** (right side up) or **inverted** (upside down)

• Image can be **reversed** ("mirror-image" – left hand looks like right and vice versa)

A plain mirror image is **virtual**, **erect**, and **reversed**
Reflection in a Sphere

- Equations are involved.
- $s'$ depends on $\alpha$

\[
\begin{align*}
\phi &= \alpha + \theta \\
\beta &= \varphi + \theta \\
\tan \alpha &= \frac{h}{s - \delta} \\
\tan \beta &= \frac{h}{s' - \delta} \\
\tan \phi &= \frac{h}{R - \delta}
\end{align*}
\]
Reflection in a Sphere

- Use **paraxial approximation**

\[ \varphi = \alpha + \theta \]
\[ \beta = \varphi + \theta \]

\[ \tan \alpha = \frac{h}{s - s'} \]
\[ \tan \beta = \frac{h}{s' - s} \]
\[ \tan \phi = \frac{h}{R - \delta} \]
Spherical Mirror Reflection

\[
\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}
\]

no \( \alpha \) dependence!
Focal Point

object is far away to the left

\[ \frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \]

\[ s' = \frac{R}{2} = f \]

focal length
Focal Point

These things are approximately true for spherical mirrors. They are exactly true for parabolic mirrors.

\[ \frac{1}{s} + \frac{1}{\infty} = \frac{2}{R} \]

\[ s = \frac{R}{2} = f \]

works in both ways
Spherical Mirror: $s > f$

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

$s > f$

$s' > 0$
Spherical Mirror: $s < f$

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

The image is virtual.

$s < f$

$s' < 0$
Spherical Mirror Magnification

The magnification $m$ of a spherical mirror can be expressed as:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

because triangles PQV and P’Q’V are similar.

$s' > 0$: image is real and inverted ($m < 0$)

$s' < 0$: image is virtual and erect ($m > 0$)
**Convex Spherical Mirror**

- **convex** = curving out
- All is exactly the same except that \( f \) (and \( R \)) is negative (concave)

![Diagram showing convex and concave spherical mirrors with labeled distances](Image)
Focal Point of Convex Mirror

provided $s > 0$, a convex mirror always forms a virtual, erect, reversed image (same as the plain mirror)

$m = 1$ for plain mirror
$m < 1$ for convex mirror
Principal rays

- Need them to find image position and magnification

- $QBQ'$: ray parallel to the axis reflects through focal point
- $QAQ'$: ray through focal point reflects parallel to the axis
- $QCQ'$: ray through the center reflects back
- $QVQ'$: ray to the vertex forms equal angles with the axis

This construction neglects aberrations
Snell’s law

• This is the basic law of refraction

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

angle of incidence not equal to angle of refraction!

what is this?

\( n \): index of refraction
Index of Refraction

• = ratio of the speed of light in the material to that in vacuum

\[ n = \frac{c}{v} \]

\( n > 1 \): light travels slower in the material than in vacuum

What changes when the light passes from one medium to another?
  * Frequency? **No**, it would imply creating/destroying waves
  * Speed? **Yes**, because the media have different \( n \)
  * Wavelength? **Yes**, because \( \lambda = \frac{v}{f} \)
Total Internal Reflection

• Snell’s law may give \( \sin \theta > 1 \) – what does it mean?

• There are always two rays: reflected and refracted

• At some angle, the refracted ray disappears

\[ n_a = n_b \sin \theta_c \]

can only happen if \( n_a < n_b \)
Fiber Optics

• Light can be transmitted along a fiber with almost no loss due to total internal reflection
  – Due to impurity of glass, the signal eventually degrades (typical rates are ~50%/km)

Widely used in communications – much higher frequency than for regular wires, therefore can transmit much more data
Refraction at a Sphere

\[ \theta_a = \alpha + \varphi \]
\[ \varphi = \beta + \theta_b \]
\[ \tan \alpha = \frac{h}{s + \delta} \]
\[ \tan \beta = \frac{h}{s' - \delta} \]
\[ \tan \phi = \frac{h}{R - \delta} \]
Refraction at a Sphere

\[ \theta_a = \alpha + \varphi \]
\[ \varphi = \beta + \theta_b \]
\[ \tan \alpha = \frac{h}{s + \delta} \]
\[ \tan \beta = \frac{h}{s' - \delta} \]
\[ \tan \phi = \frac{h}{R - \delta} \]
Refraction at a Sphere

- Use Snell’s law

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

\[ \theta_a = \alpha + \varphi \]

\[ \varphi - \beta = \theta_b \]

\[ n_a (\alpha + \varphi) = n_b (\varphi - \beta) \]

\[ \frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \]

magnification:
\[ m = -\frac{n_a s'}{n_b s} \]
Thin Lens

- Lens = an optical system with two refracting surfaces

Thin lens:

\[ s_2 \approx -s_1' \]
Thin Lens Equation

\[
\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}, \quad \frac{n_b}{s_2} + \frac{n_a}{s_2'} = \frac{n_a - n_b}{R_2}
\]

assumptions: \( n_a = 1, \quad n_b = n, \quad s_2 = -s_1' \)

\[
\frac{1}{s_1} + \frac{1}{s_2'} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}
\]