Chapter 23

Geometric Optics

Light

- What is light? Waves or particles?

both

- Geometric optics: light travels in straight-line paths called rays
  - This is true if typical distances are much larger than the wavelength

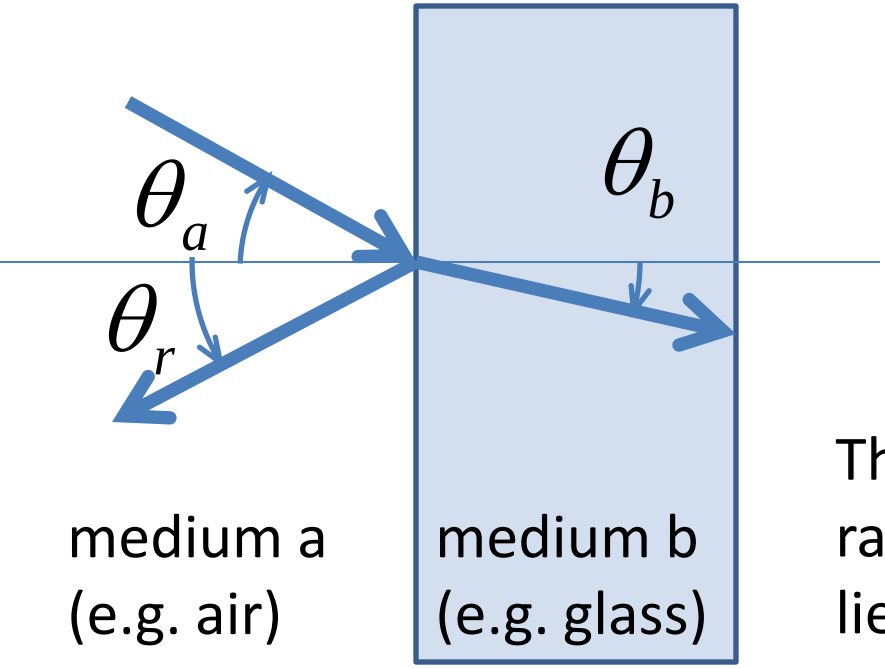
What it is about

- Phenomena addressed by geometric optics:
  - Propagation: light goes in straight lines
  - Reflection: angle of incidence = angle of reflection
  - Refraction: Snell’s law
- Phenomena not addressed by geometric optics:
  - Polarization
  - Diffraction
  - Interference
  - Photoelectric effect

Electromagnetic theory

Quantum mechanics

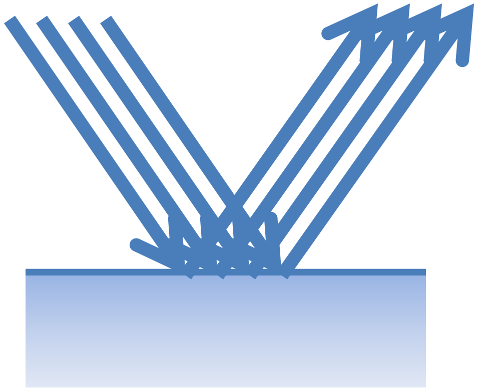
Reflection and Refraction



θ_a	incident angle
θ_r	reflection angle
θ_b	refraction angle

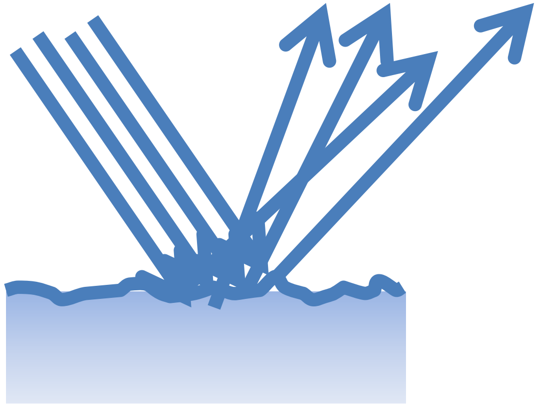
The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane

Specular and Diffusive Reflection



we’ll discuss this one

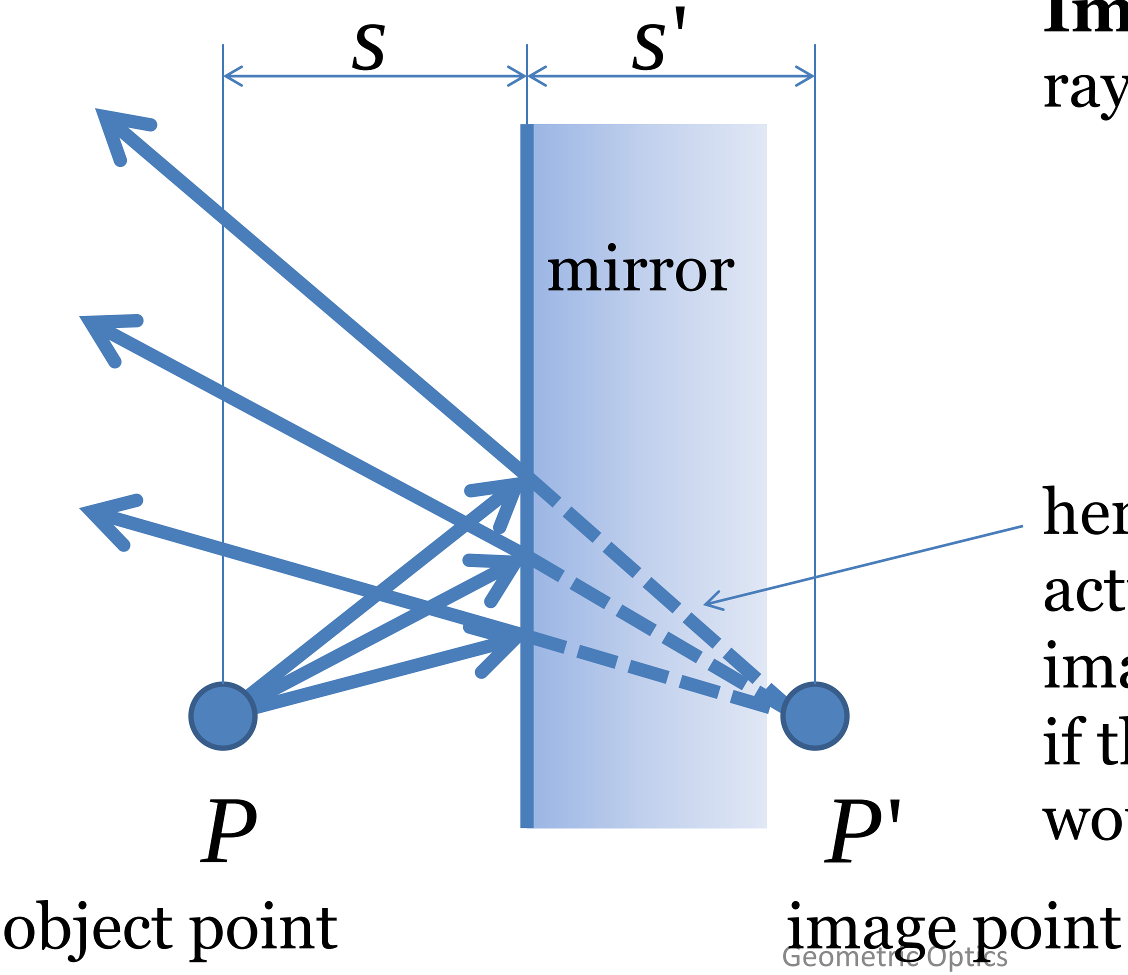
specular reflection: smooth interface, definite reflection angle



diffusive reflection: rough interface, scattered reflection

Image

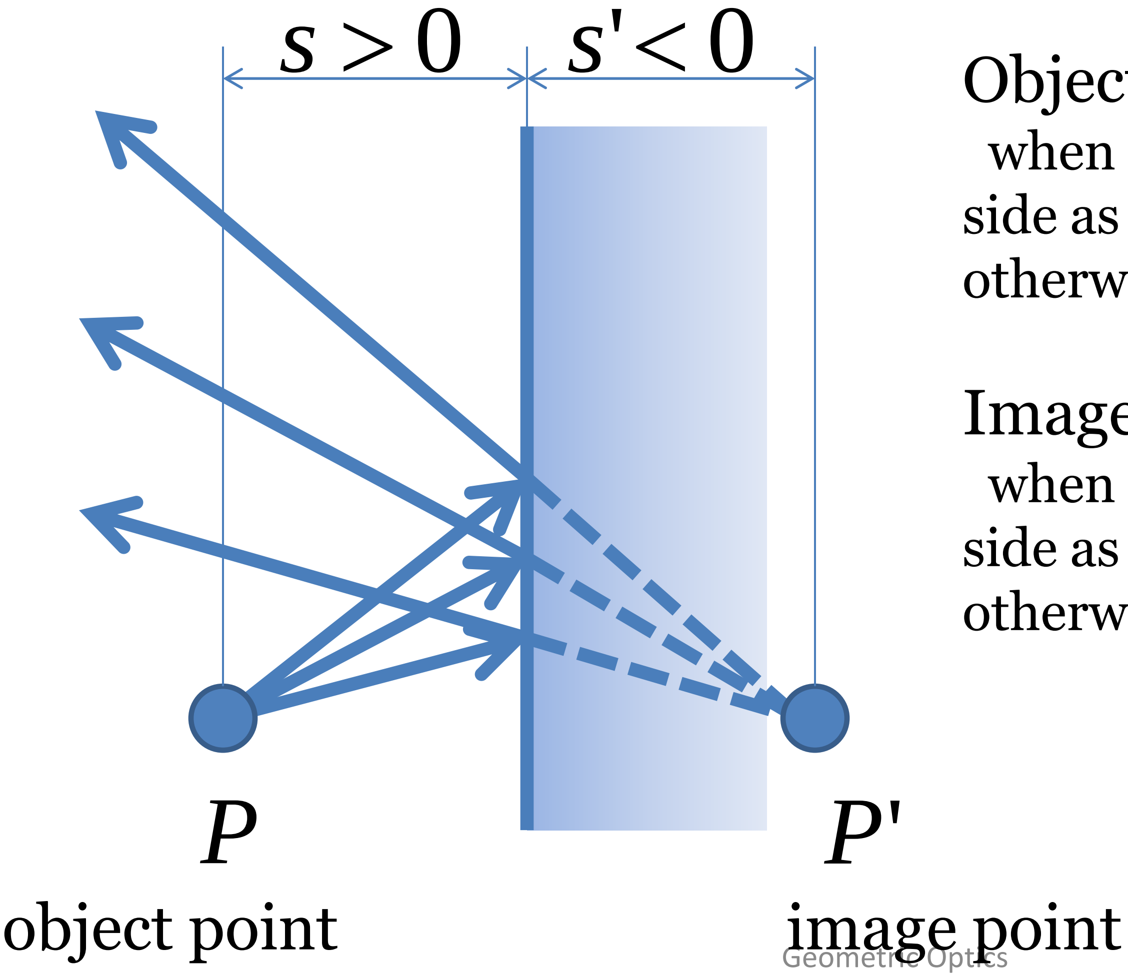
Object point: where the rays actually come from
Image point: where the rays appear to come from



s: **object distance**
s’: **image distance**

here outgoing rays do not actually come from P’ → image is **virtual**
if they did, the image would be **real**

Sign Rules for Distances



Object distance:

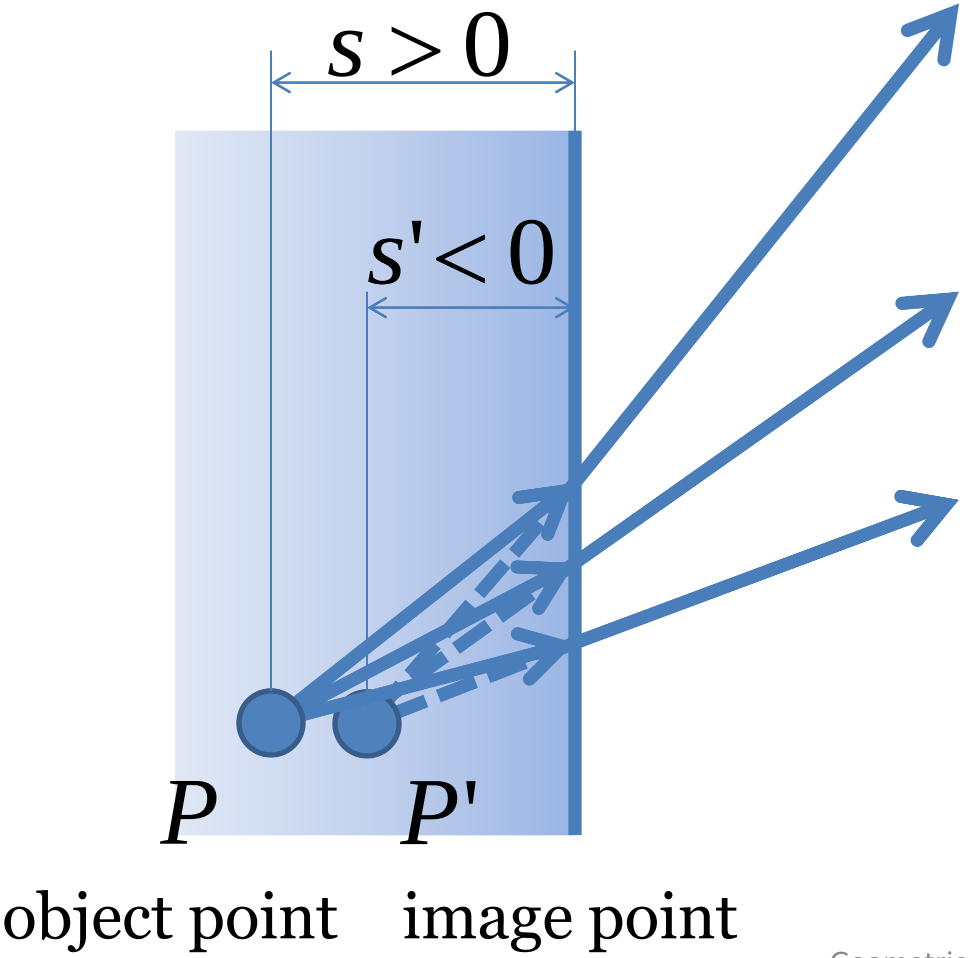
when the object is on the same side as the incoming light, $s>0$, otherwise, $s<0$

Image distance:

when the image is on the same side as the outgoing light, $s'>0$, otherwise, $s'<0$

mirror

Sign Rules for Distances



Object distance:

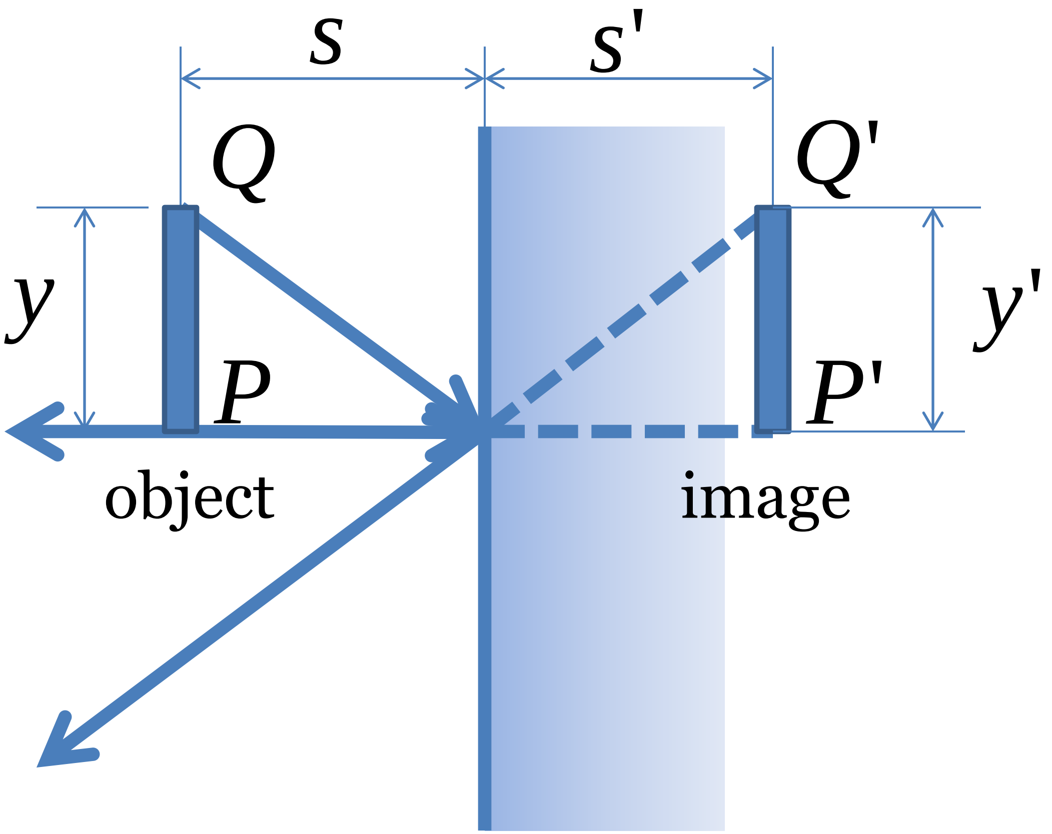
when the object is on the same side as the incoming light, $s>0$, otherwise, $s<0$

Image distance:

when the image is on the same side as the outgoing light, $s'>0$, otherwise, $s'<0$

refracting interface

Lateral Magnification



$$m = \frac{y'}{y}$$

mirror:

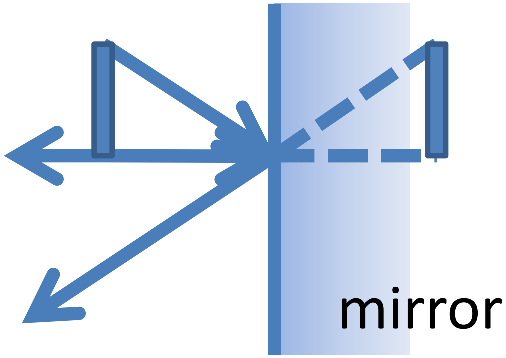
$$s = -s'$$

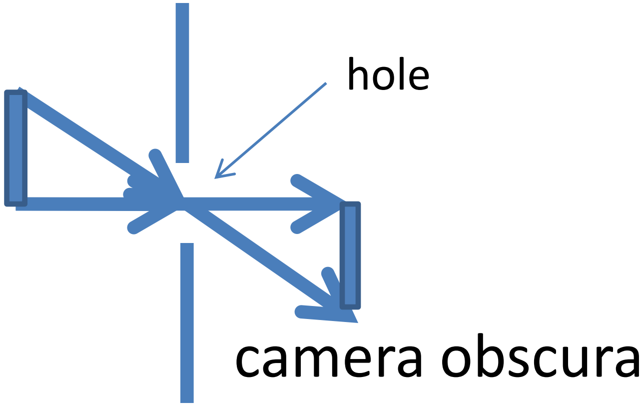
$$m = 1$$

need at least two points (P,Q) to figure it out

Inverted and Reversed Images

- Image can be **erect** (right side up) or **inverted** (upside down)



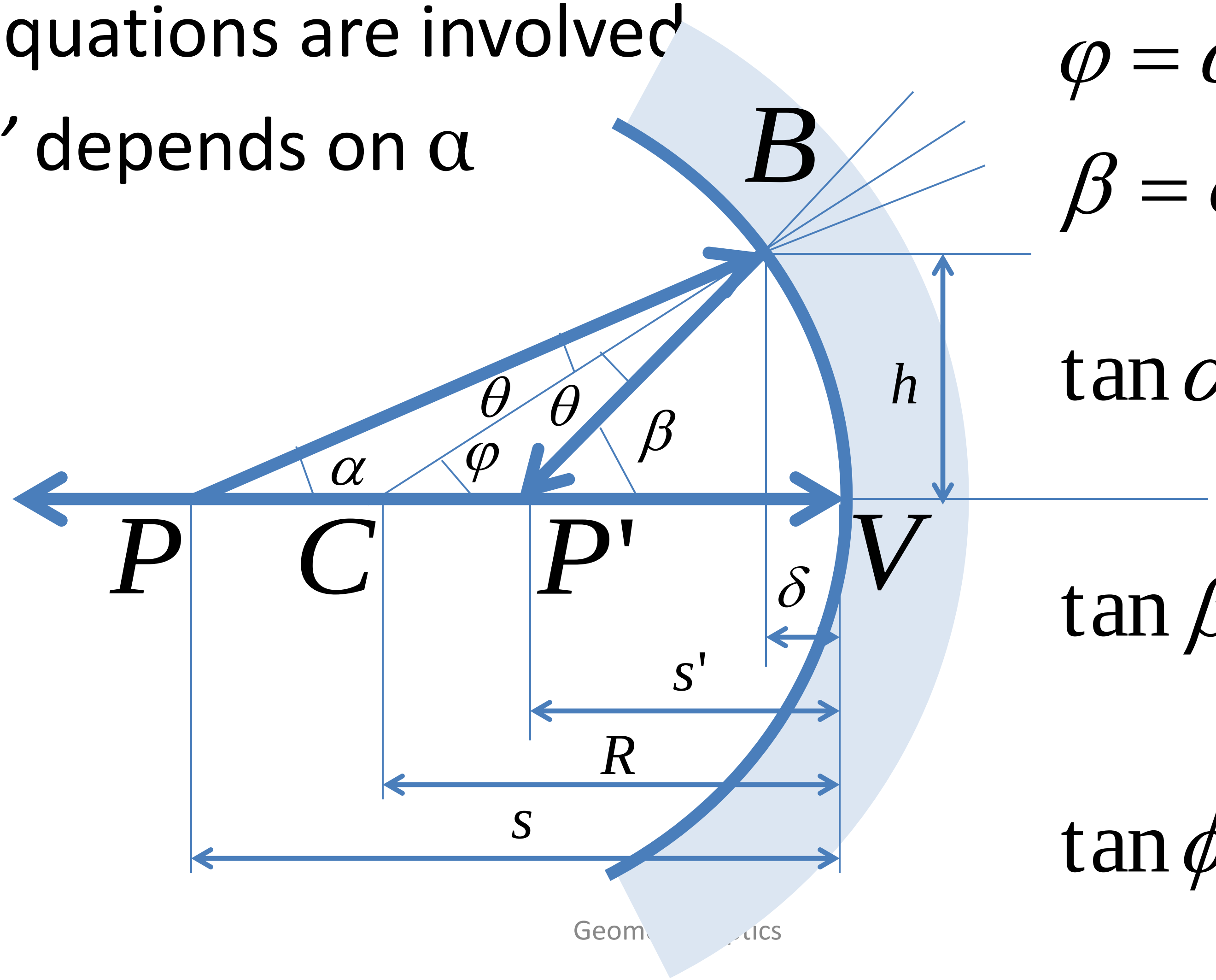


- Image can be **reversed** (“mirror-image” – left hand looks like right and vice versa)

a plain mirror image is **virtual**, **erect**, and **reversed**

Reflection in a Sphere

- Equations are involved
- *s'* depends on α



$$\varphi = \alpha + \theta$$

$$\beta = \varphi + \theta$$

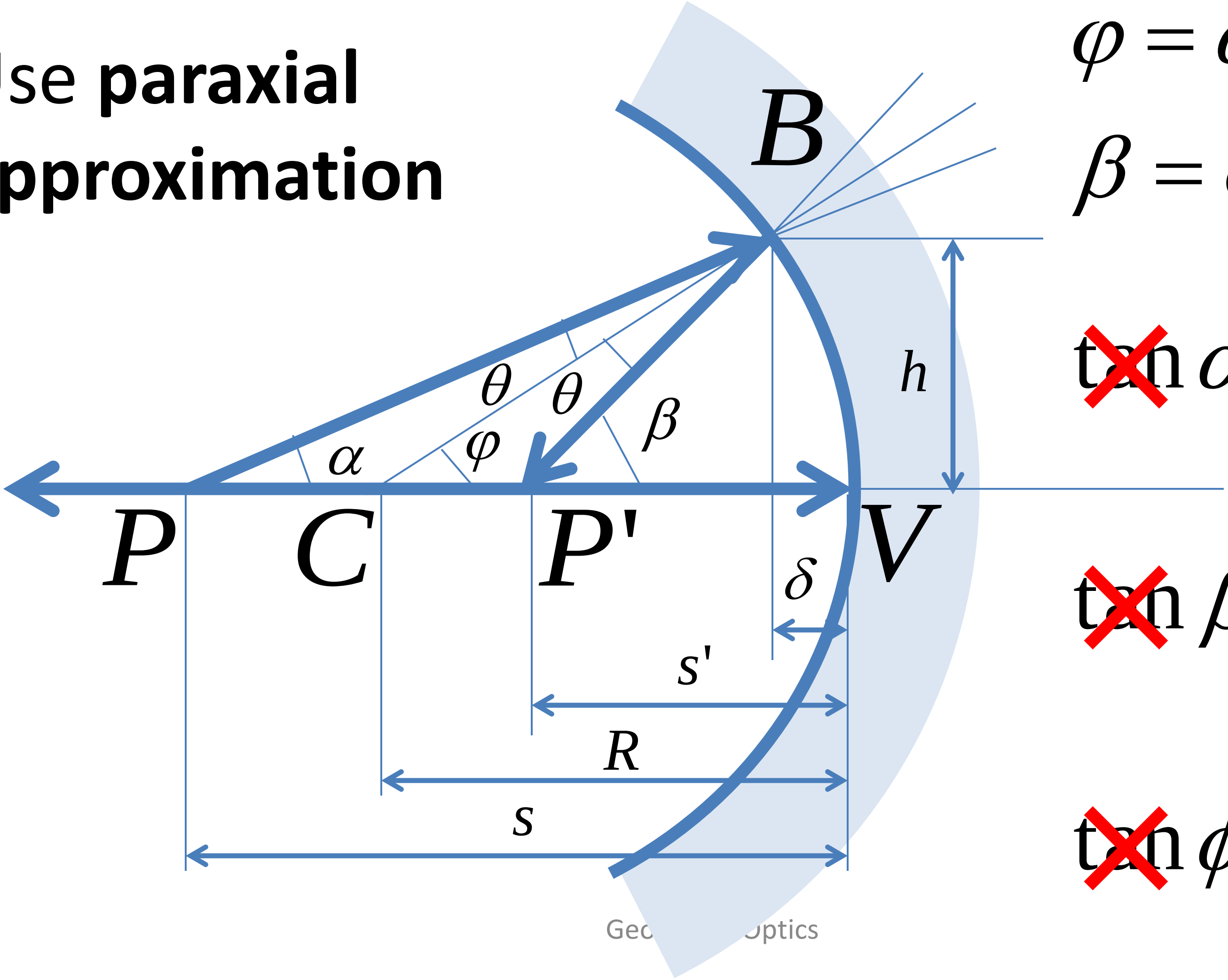
$$\tan\alpha = \frac{h}{s-\delta}$$

$$\tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

Reflection in a Sphere

- Use **paraxial approximation**



$$\varphi = \alpha + \theta$$

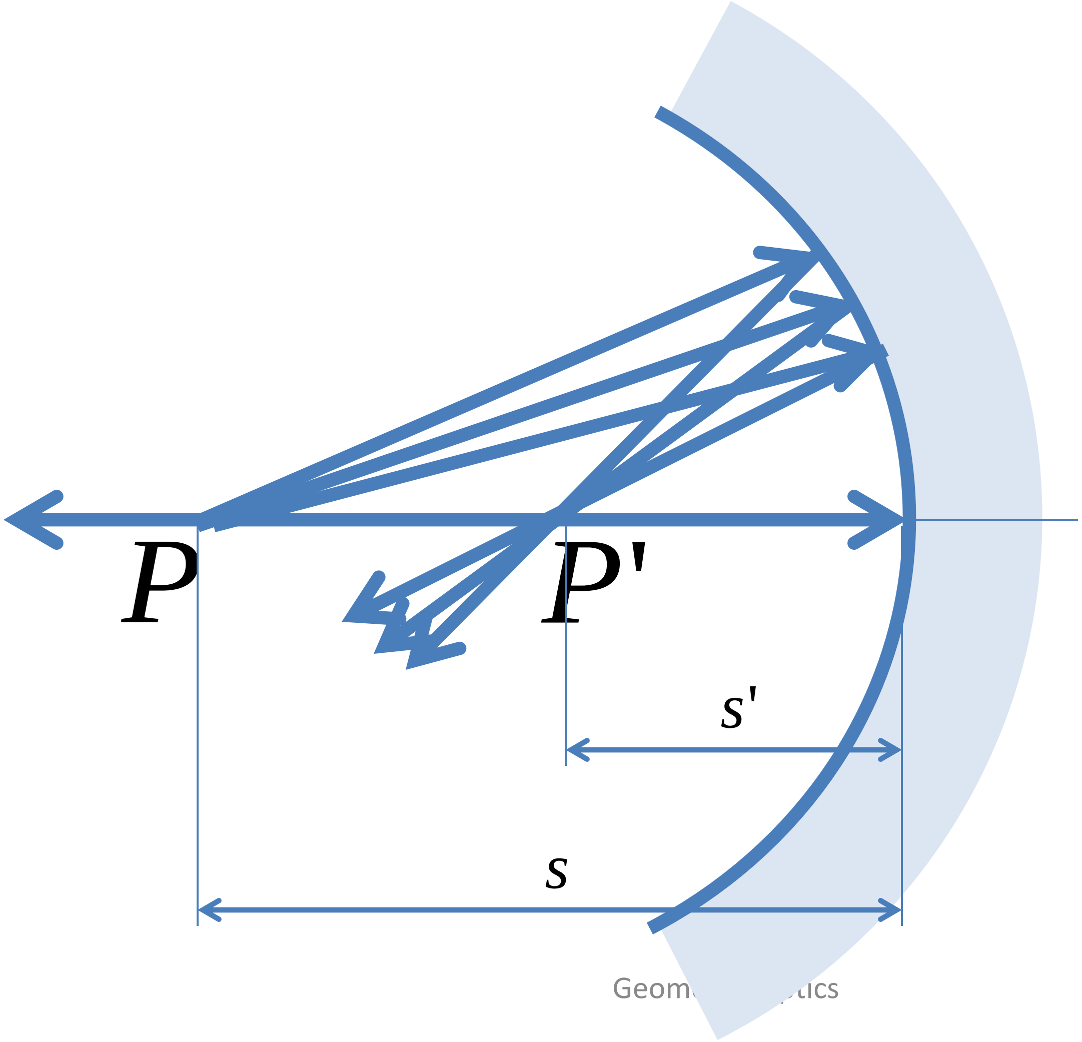
$$\beta = \varphi + \theta$$

$$\cancel{\tan}\,\alpha = \frac{h}{s - \cancel{\delta}}$$

$$\cancel{\tan}\,\beta = \frac{h}{s' - \cancel{\delta}}$$

$$\cancel{\tan}\,\phi = \frac{h}{R - \cancel{\delta}}$$

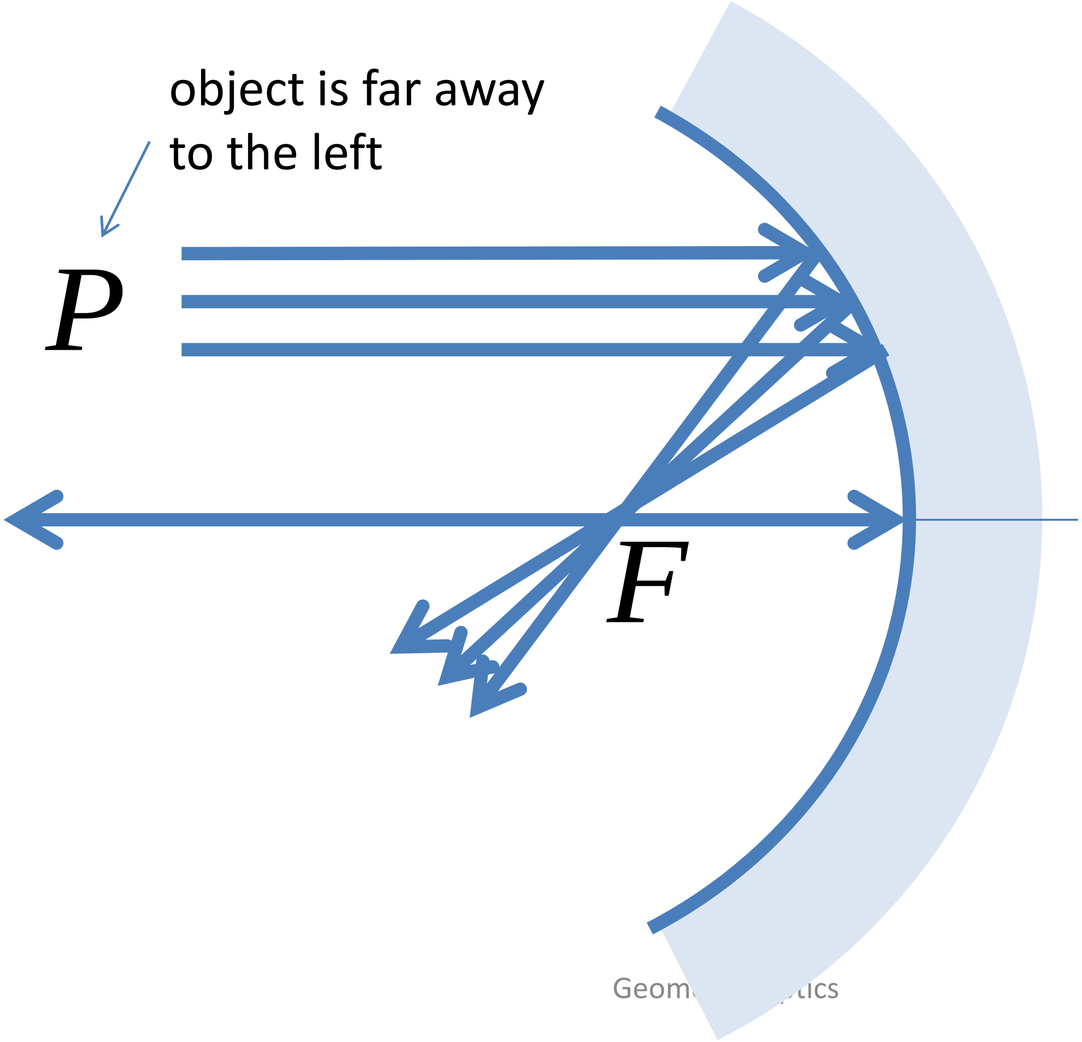
Spherical Mirror Reflection



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

no α dependence!

Focal Point



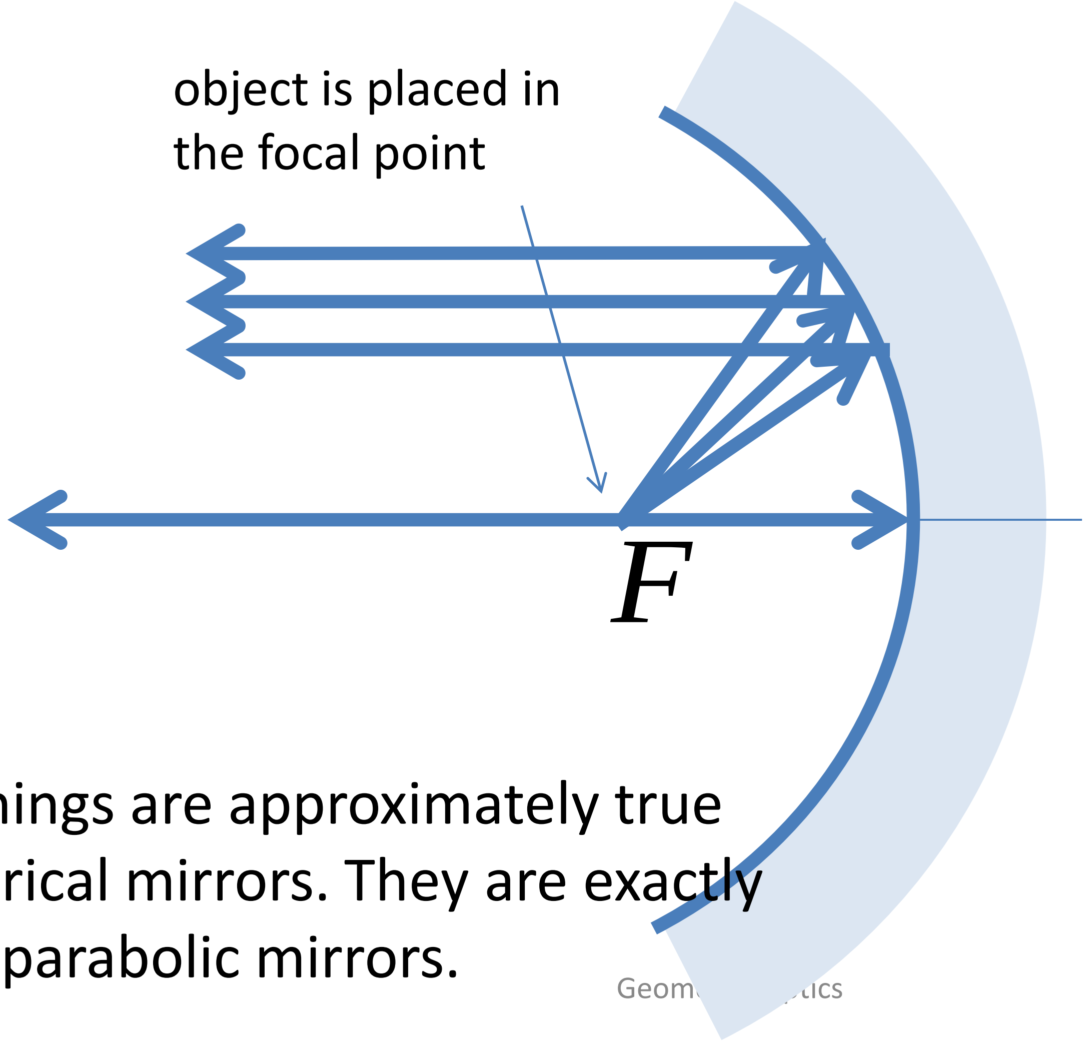
object is far away to the left

$$\frac{1}{\infty}+\frac{1}{s'}=\frac{2}{R}$$

$$s'=\frac{R}{2}=f$$

focal length

Focal Point



object is placed in the focal point

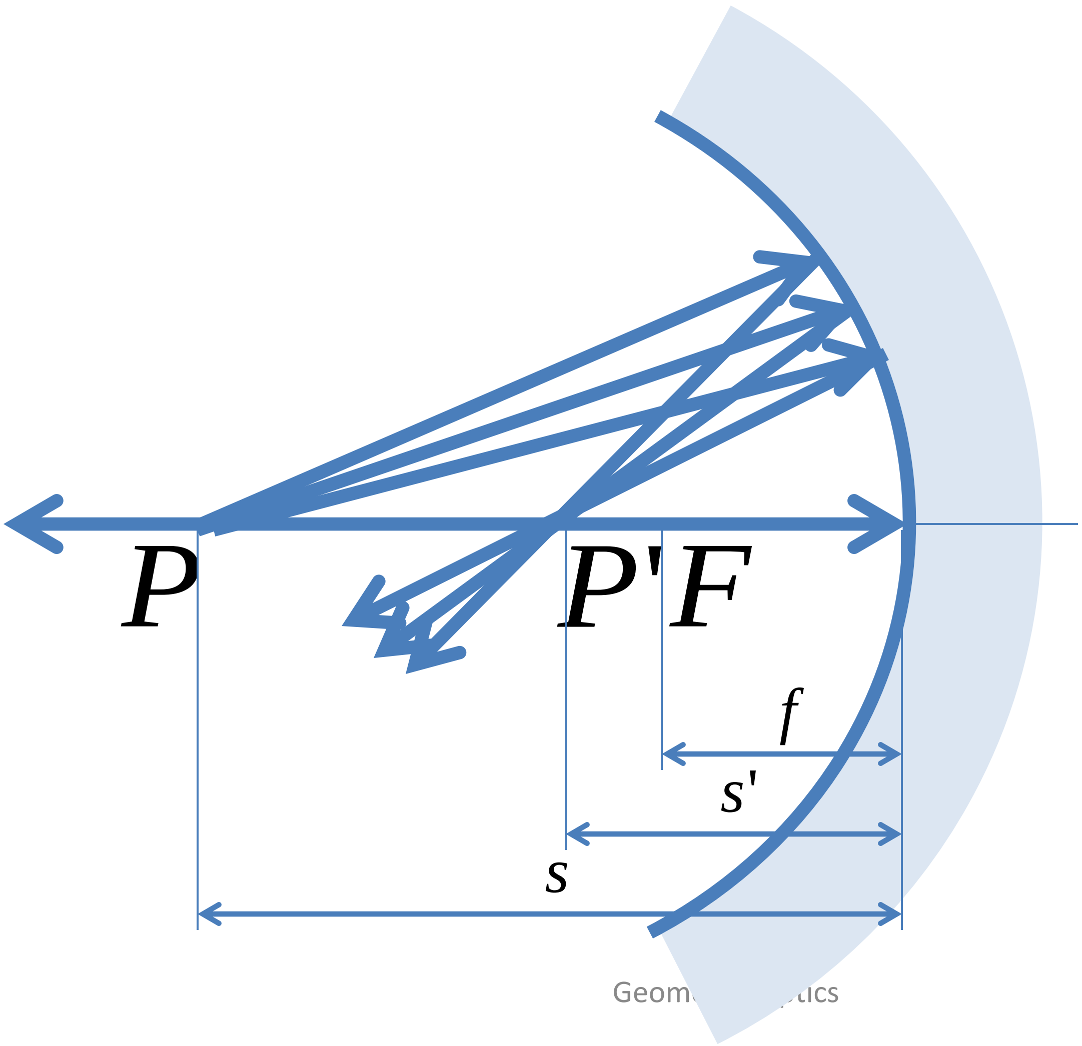
$$\frac{1}{s} + \frac{1}{\infty} = \frac{2}{R}$$

$$s = \frac{R}{2} = f$$

works in both ways

These things are approximately true for spherical mirrors. They are exactly true for parabolic mirrors.

Spherical Mirror: *s>f*

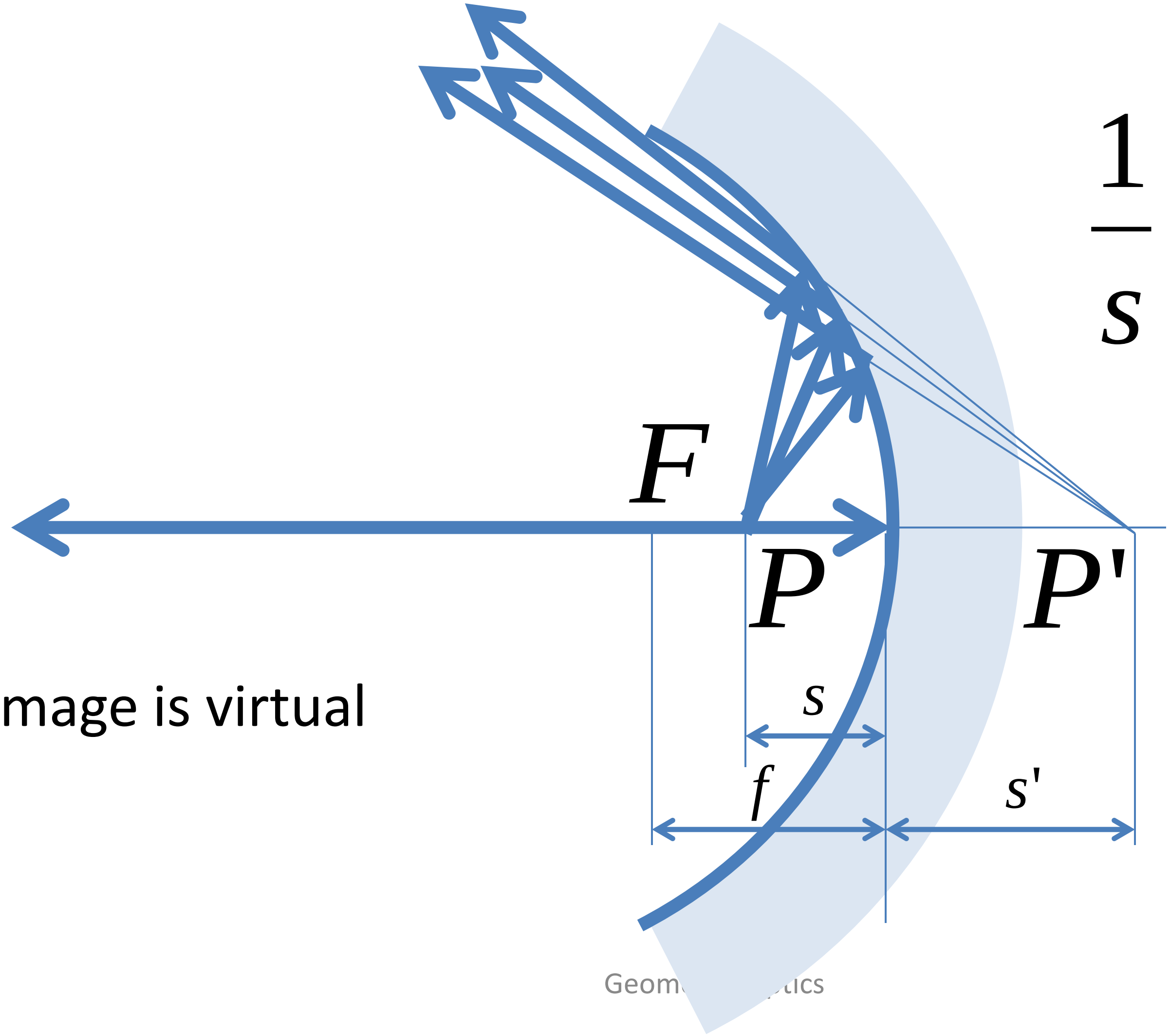


$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$s>f$$

$$s'>0$$

Spherical Mirror: *s<f*



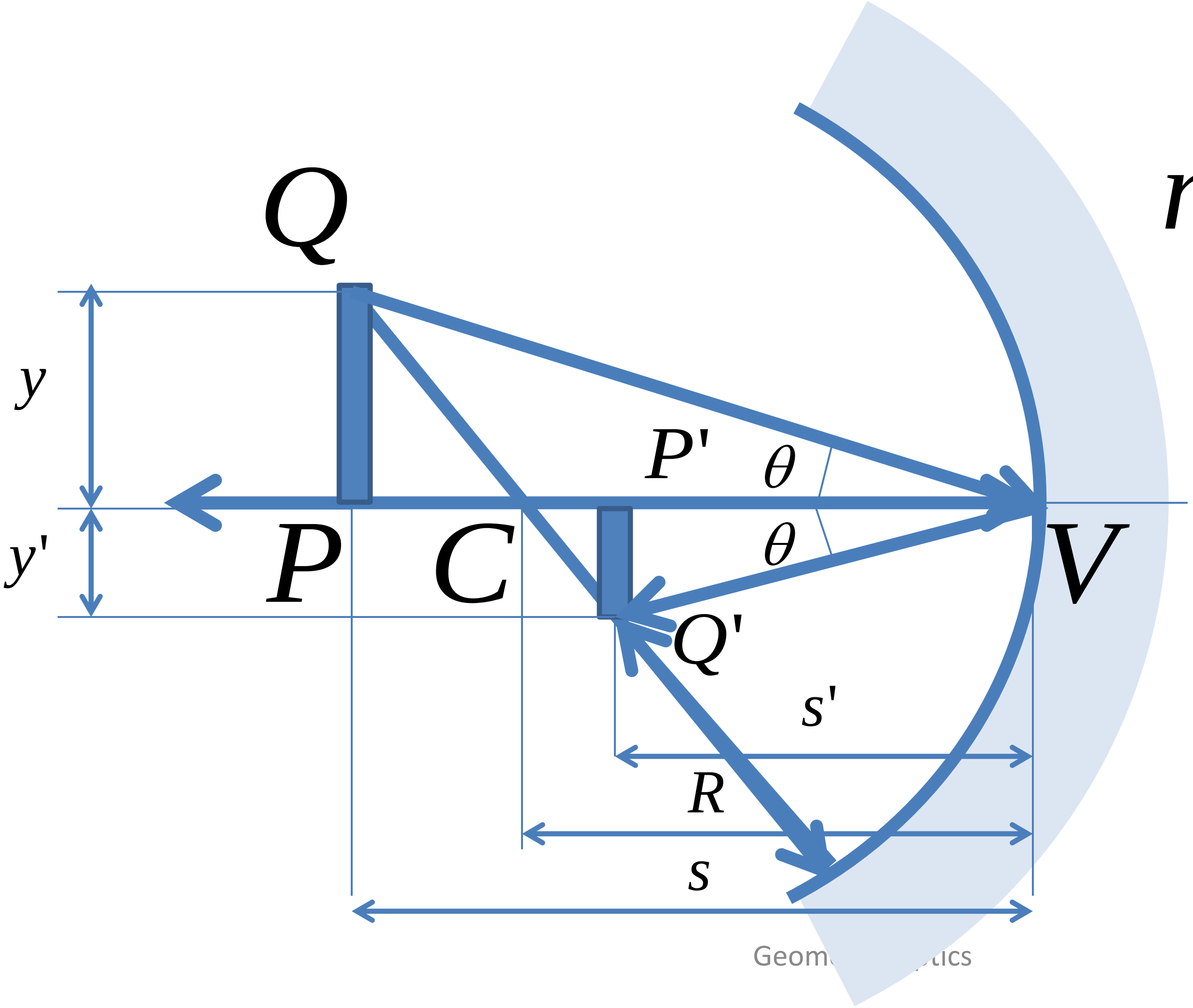
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$s<f$$

$$s'<0$$

image is virtual

Spherical Mirror Magnification



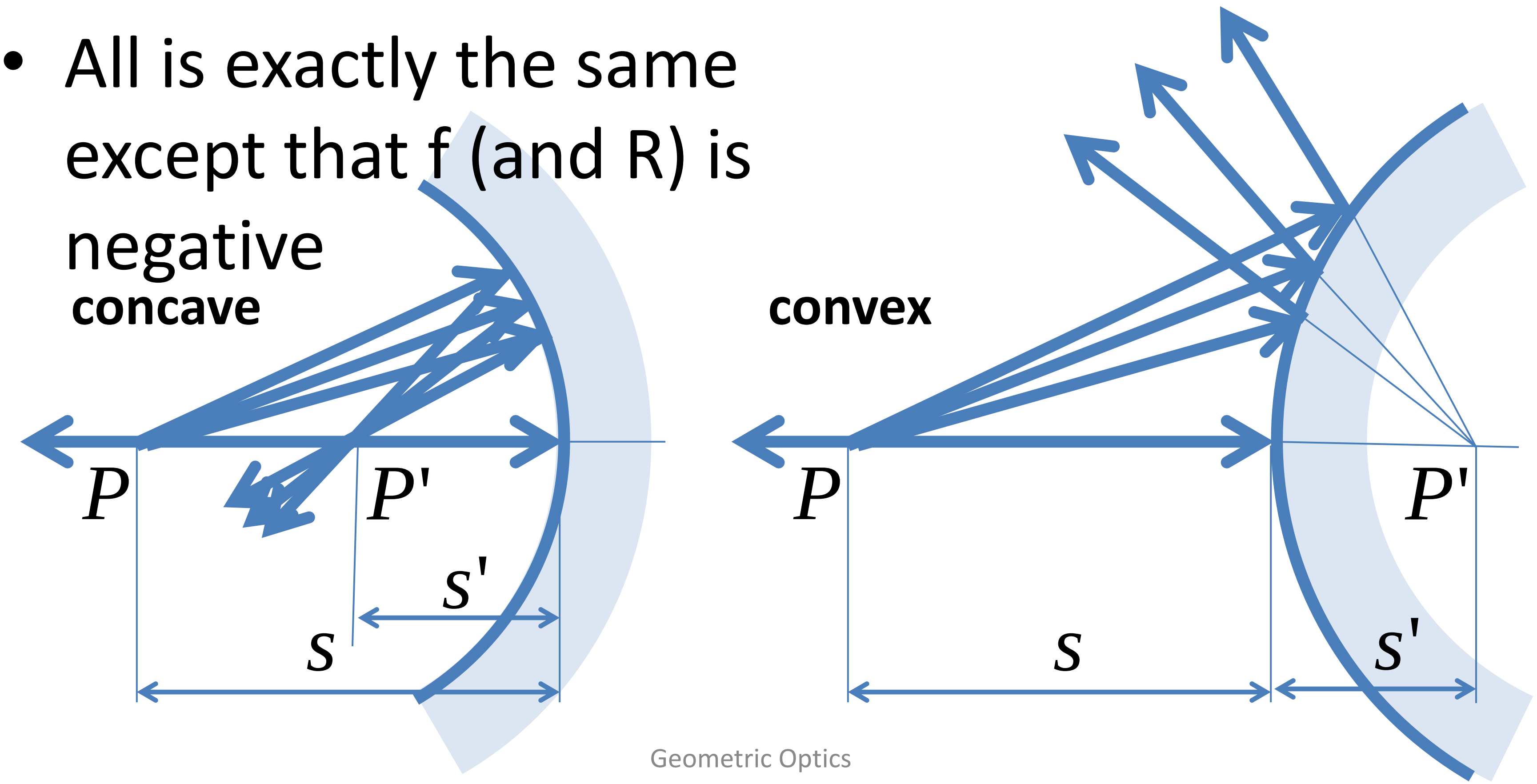
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

because triangles PQV and P’Q’V are similar

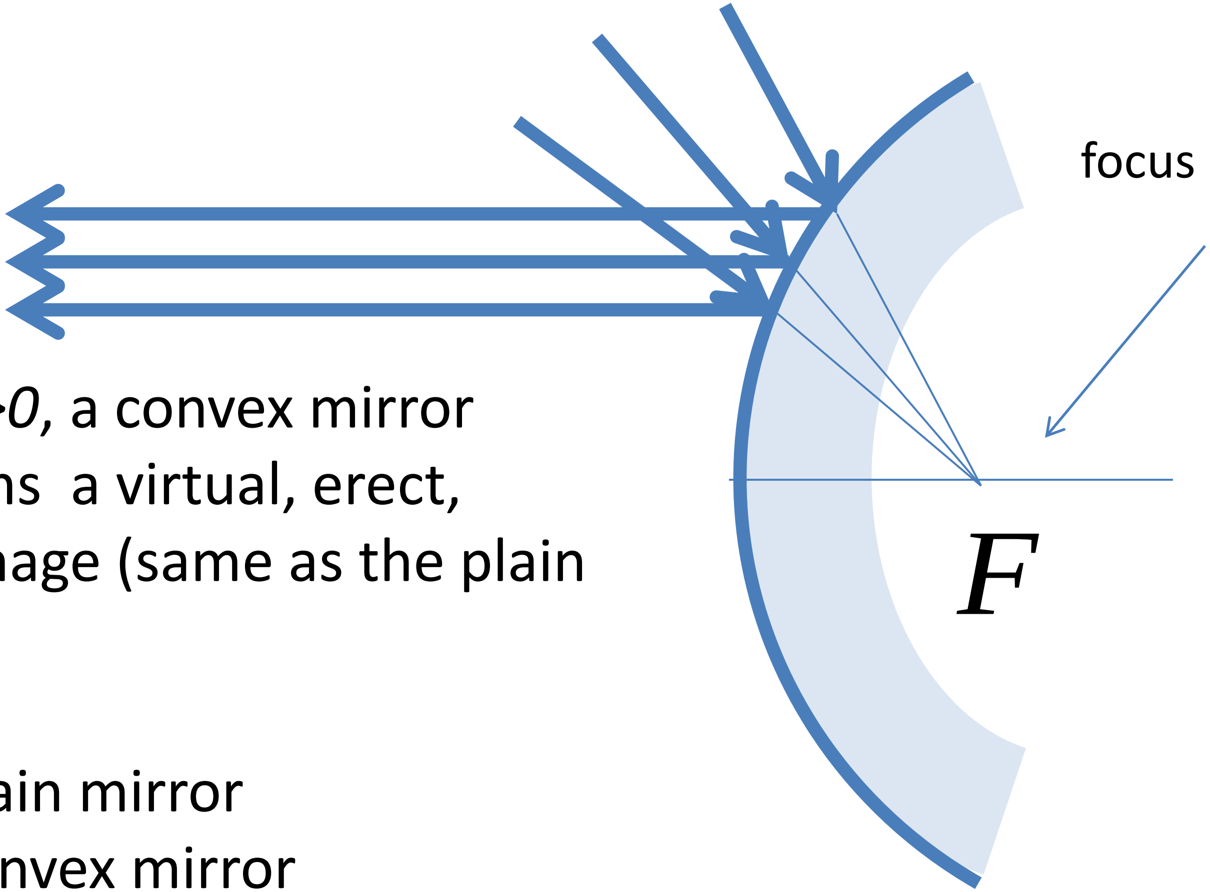
s’>0: image is real and inverted (*m<0*)
s’<0: image is virtual and erect (*m>0*)

Convex Spherical Mirror

- **convex** = curving out
- All is exactly the same except that f (and R) is negative



Focal Point of Convex Mirror



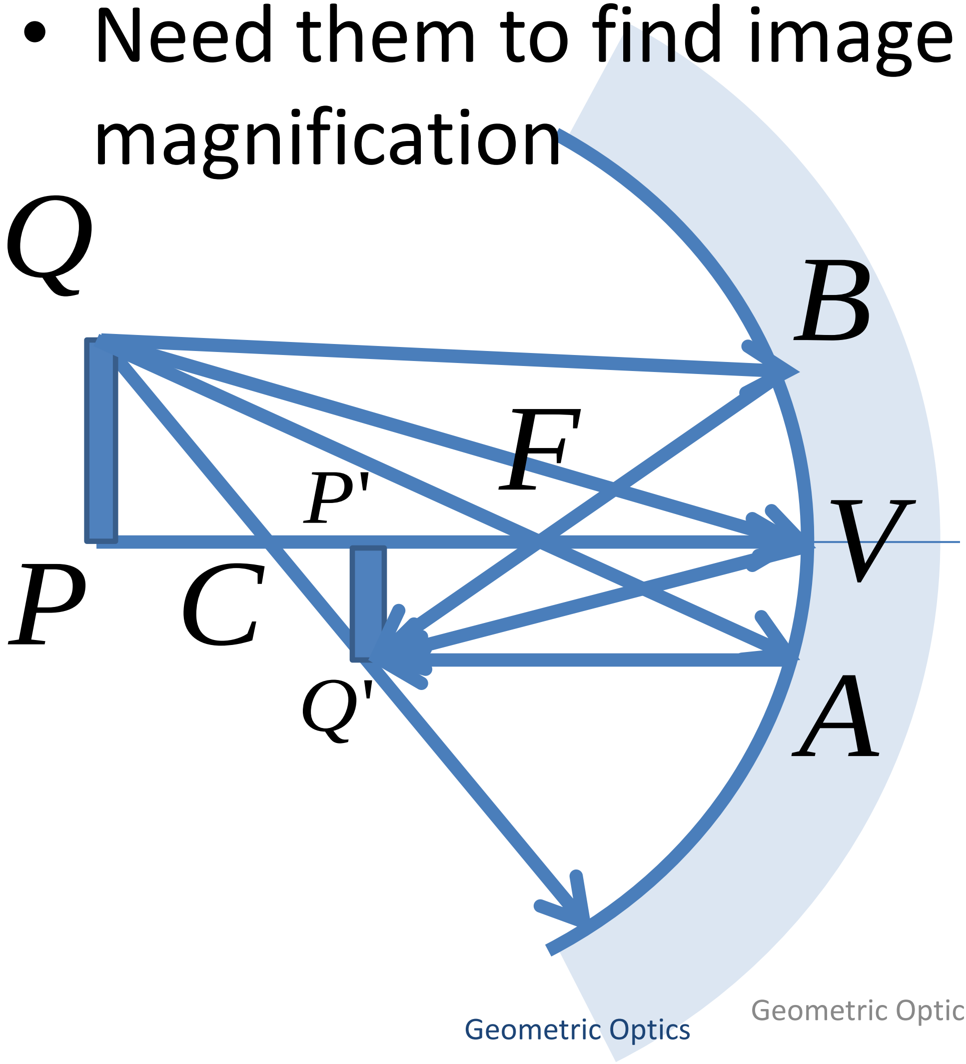
focus is virtual

provided *s>0*, a convex mirror always forms a virtual, erect, reversed image (same as the plain mirror)

m=1 for plain mirror
m<1 for convex mirror

Principal rays

- Need them to find image position and magnification

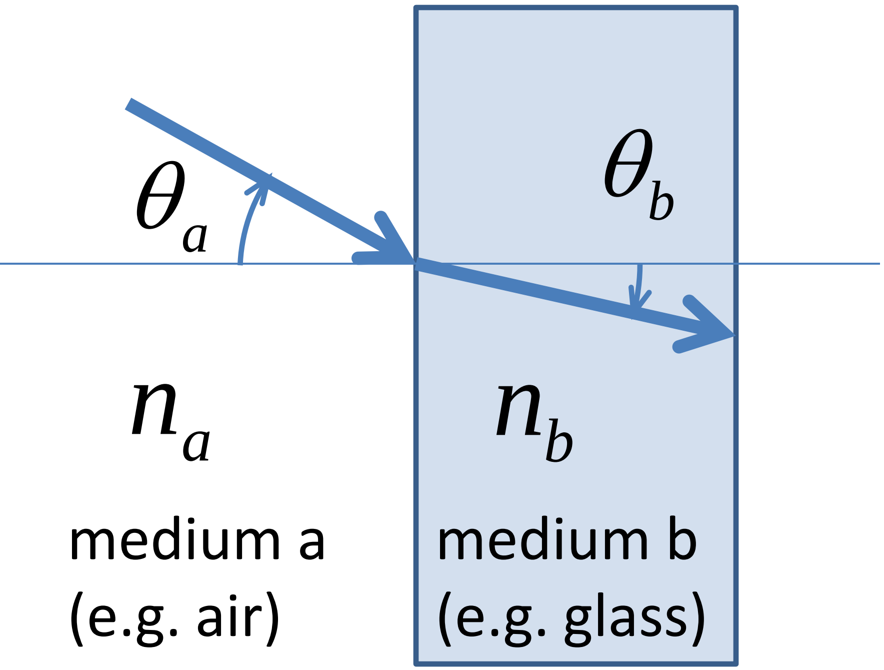


- *QBQ′*: ray parallel to the axis reflects through focal point
- *QAQ′*: ray through focal point reflects parallel to the axis
- *QCQ′*: ray through the center reflects back
- *QVQ′*: ray to the vertex forms equal angles with the axis

this construction neglects aberrations

Snell’s law

- This is the basic law of refraction



medium a
(e.g. air)

medium b
(e.g. glass)

angle of incidence not equal to
angle of refraction!

$$n_a \sin\theta_a = n_b \sin\theta_b$$

n: index of refraction
what is this?

Index of Refraction

- = ratio of the speed of light in the material to that in vacuum

$$n = \frac{c}{v}$$

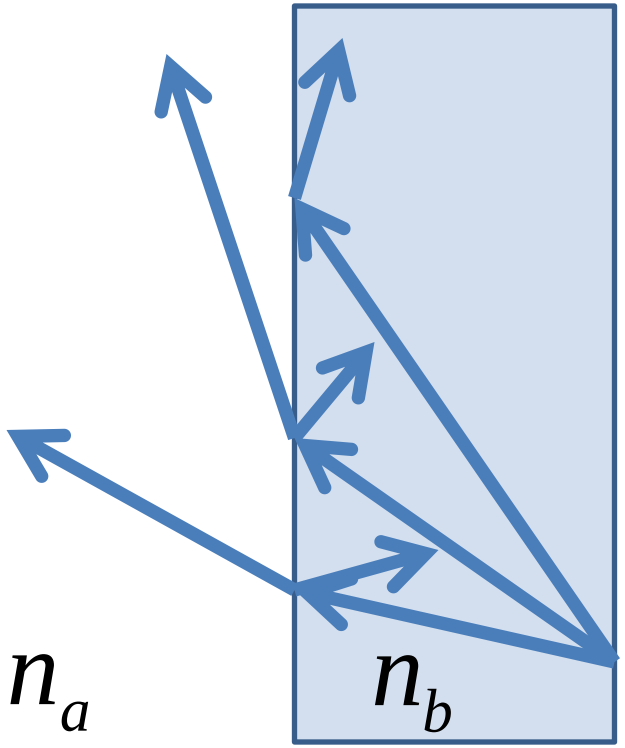
n>1: light travels slower in the material than in vacuum

What changes when the light passes from one medium to another?

* Frequency ? **No**, it would imply creating/destroying waves
* Speed? **Yes**, because the media have different *n*
* Wavelength? **Yes**, because $\lambda = v/f$

Total Internal Reflection

- Snell's law may give $\sin\theta > 1$ – what does it mean?



- There are always two rays: reflected and refracted
- At some angle, the refracted ray disappears

$$n_a = n_b \sin\theta_C$$

critical angle

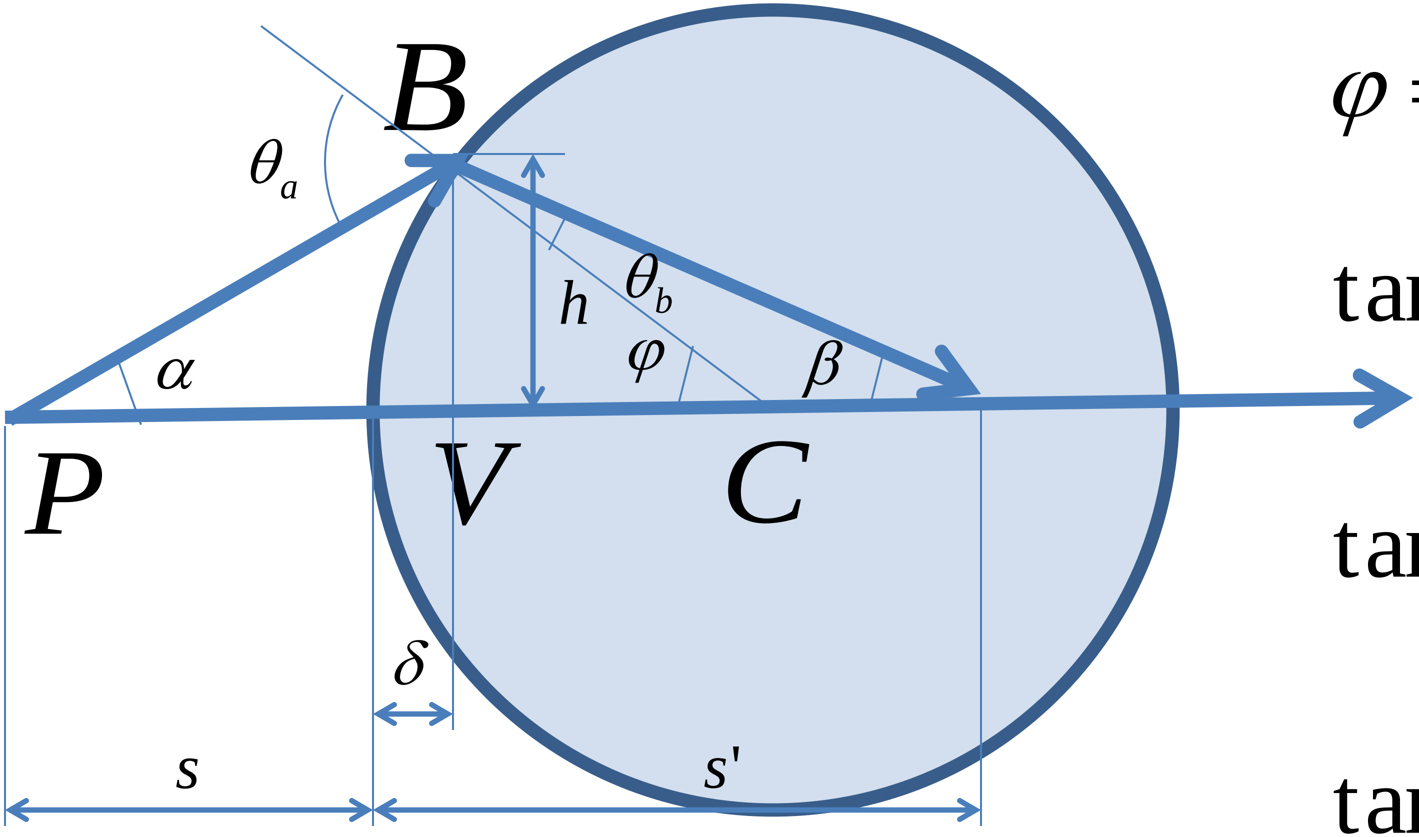
can only happen if $n_a < n_b$

Fiber Optics

- Light can be transmitted along a fiber with almost no loss due to total internal reflection
  - Due to impurity of glass, the signal eventually degrades (typical rates are ~50%/km)

Widely used in communications – much higher frequency than for regular wires, therefore can transmit much more data

Refraction at a Sphere



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$$\varphi = \beta + \theta_b$$

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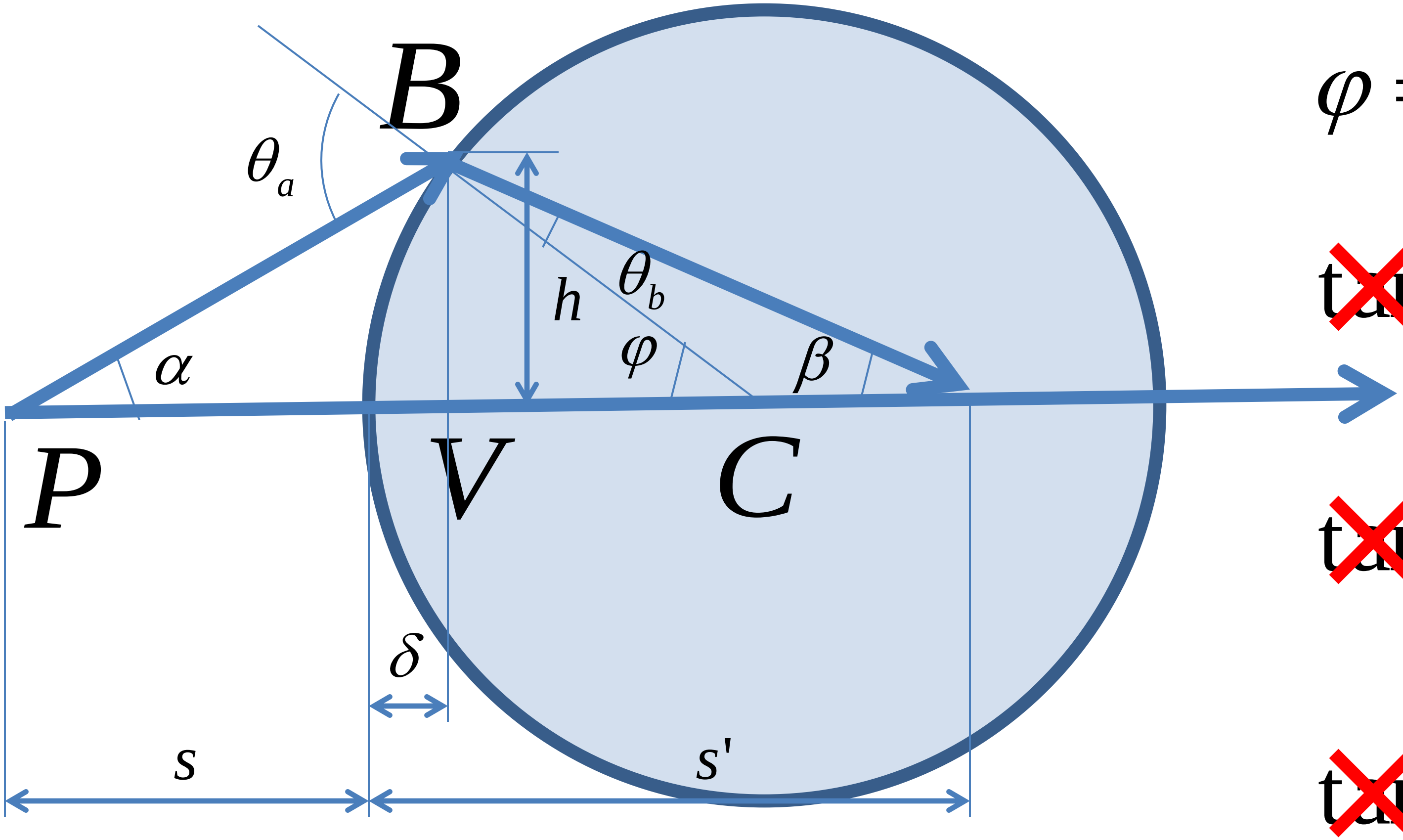
Refraction at a Sphere



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$$\cancel{\tan}\,\phi = \frac{h}{R-\cancel{\delta}}$$

Refraction at a Sphere

- Use Snell's law

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$\theta_a = \alpha + \varphi$$

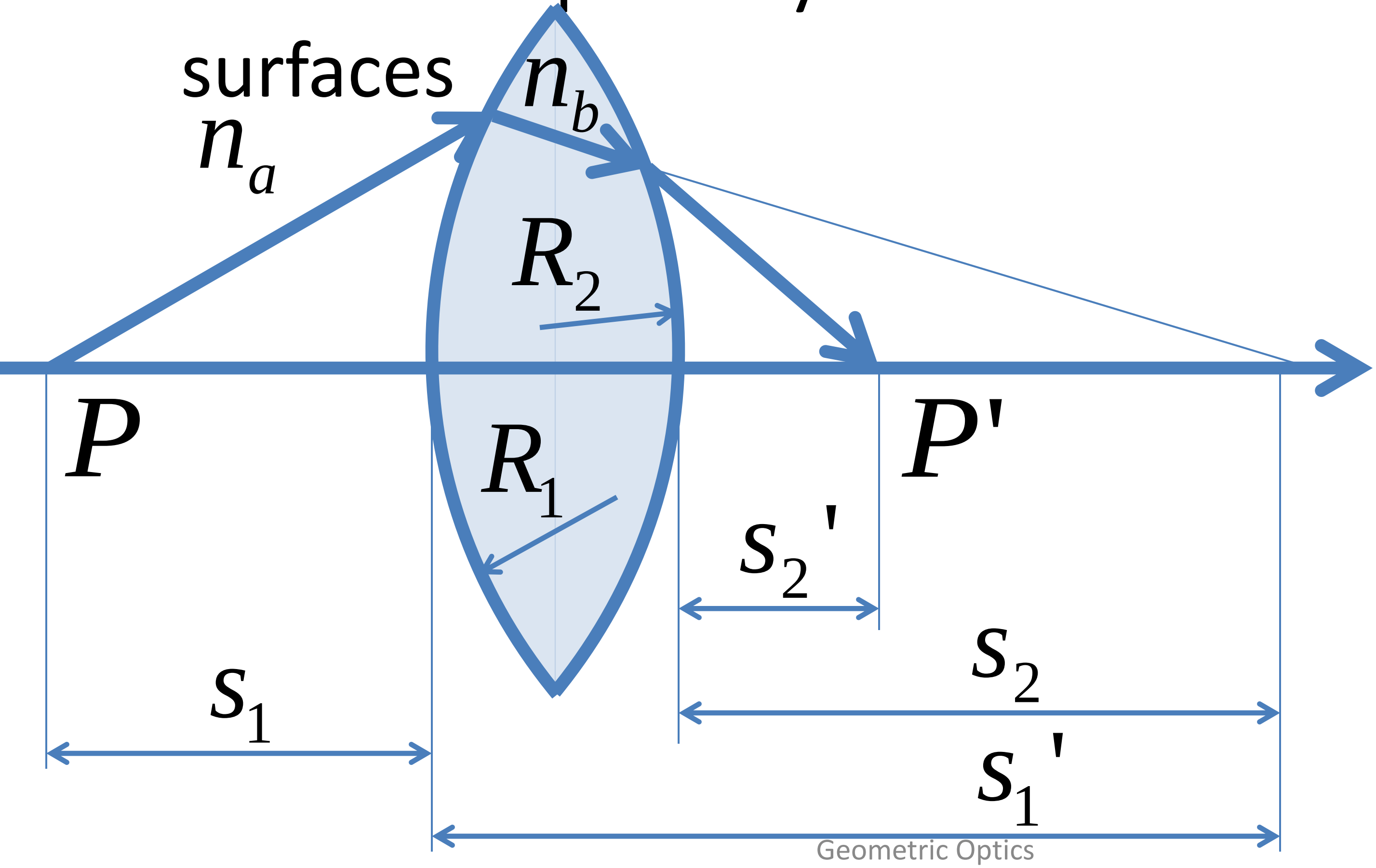
$$\varphi - \beta = \theta_b$$

$$n_a(\alpha + \varphi) = n_b(\varphi - \beta)$$

magnification: $$m = -\frac{n_a s'}{n_b s}$$

Thin Lens

- Lens = an optical system with two refracting surfaces



Thin lens:

$$s_2 \approx -s_1'$$

Thin Lens Equation

$$\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1}, \quad \frac{n_b}{s_2}+\frac{n_a}{s_2'}=\frac{n_a-n_b}{R_2}$$

assumptions: $n_a=1, \quad n_b=n, \quad s_2=-s_1'$

$$\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$