# Chapter 23

Geometric Optics

# Light

- What is light? Waves or particles?

  both

- Geometric optics: light travels in straight-line paths called rays
  - This is true if typical distances are much larger than the wavelength

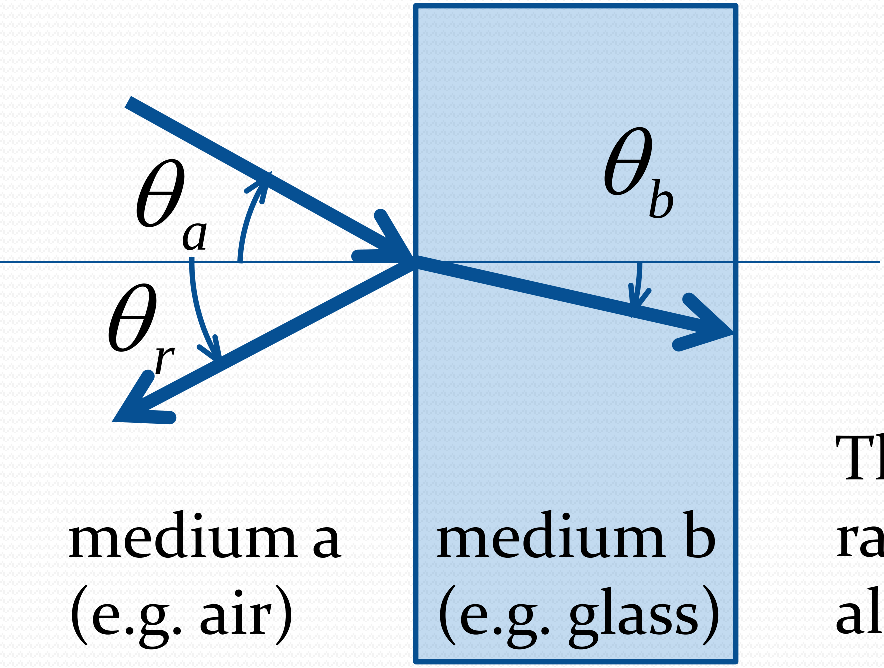
# What it is about

- Phenomena addressed by geometric optics:
  - Propagation: light goes in straight lines
  - Reflection: angle of incidence = angle of reflection
  - Refraction: Snell's law
- Phenomena not addressed by geometric optics:
  - Polarization
  - Diffraction
  - Interference
  - Photoelectric effect

Polarization, Diffraction, Interference → Electromagnetic theory

Photoelectric effect → Quantum mechanics

# Reflection and Refraction



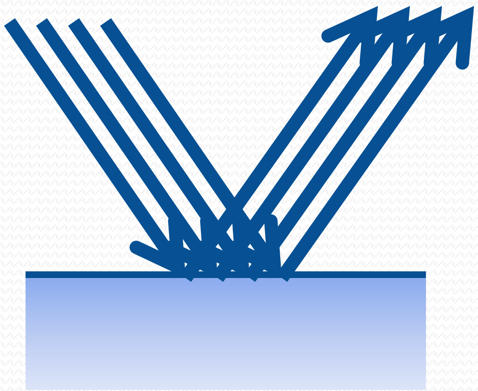

$\theta_a$ incident angle

$\theta_r$ reflection angle

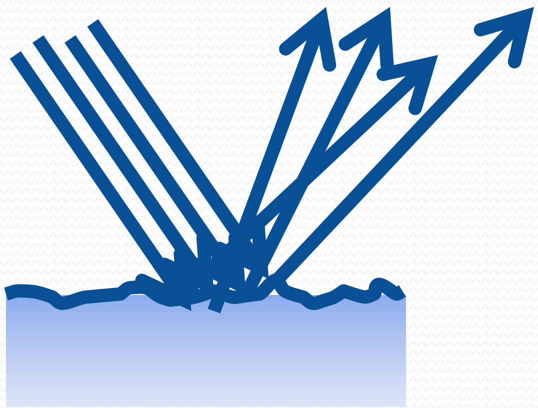
$\theta_b$ refraction angle

The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane

# Specular and Diffusive Reflection



we'll discuss
this one

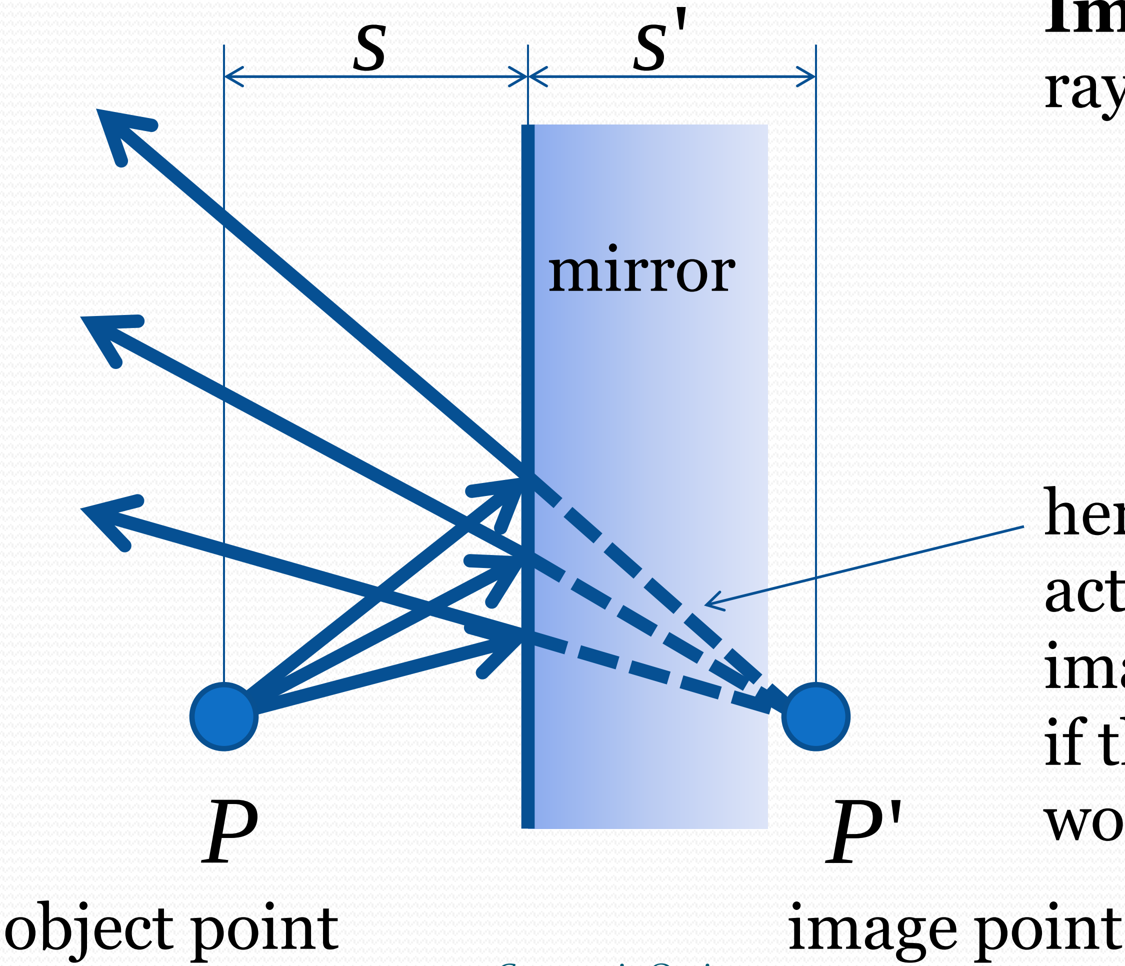


**specular** reflection:
smooth interface,
definite reflection angle

**diffusive** reflection:
rough interface,
scattered reflection

# Image

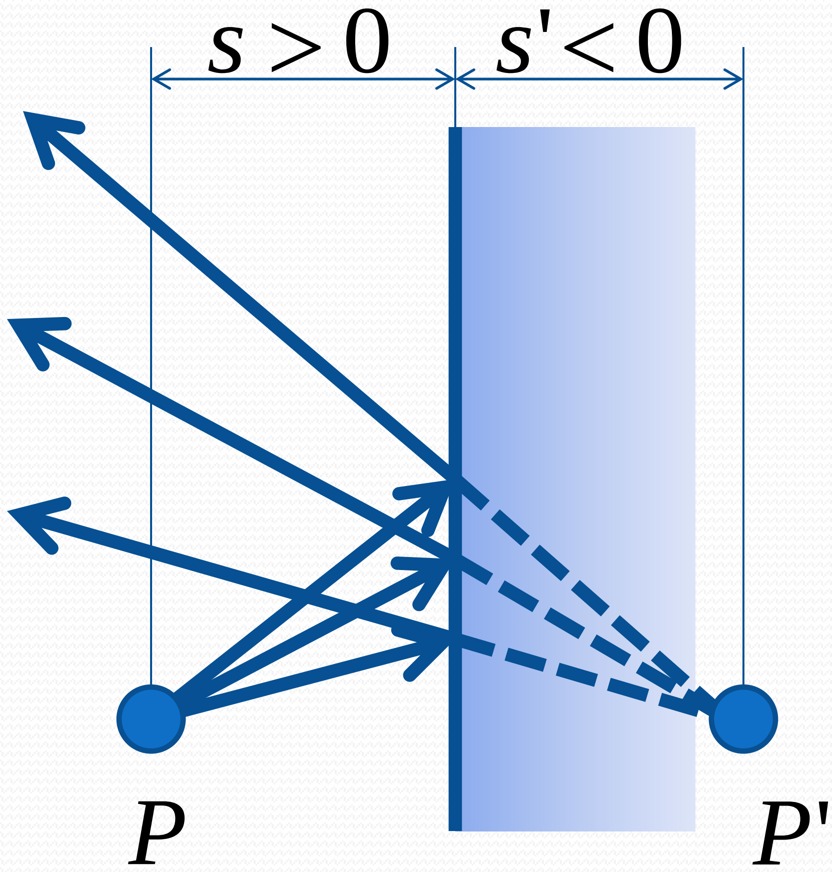
**Object point**: where the rays actually come from
**Image point**: where the rays appear to come from




*s*: **object distance**
*s'*: **image distance**

here outgoing rays do not actually come from P' → image is **virtual**
if they did, the image would be **real**

# Sign Rules for Distances




object point

image point

**Object distance:**

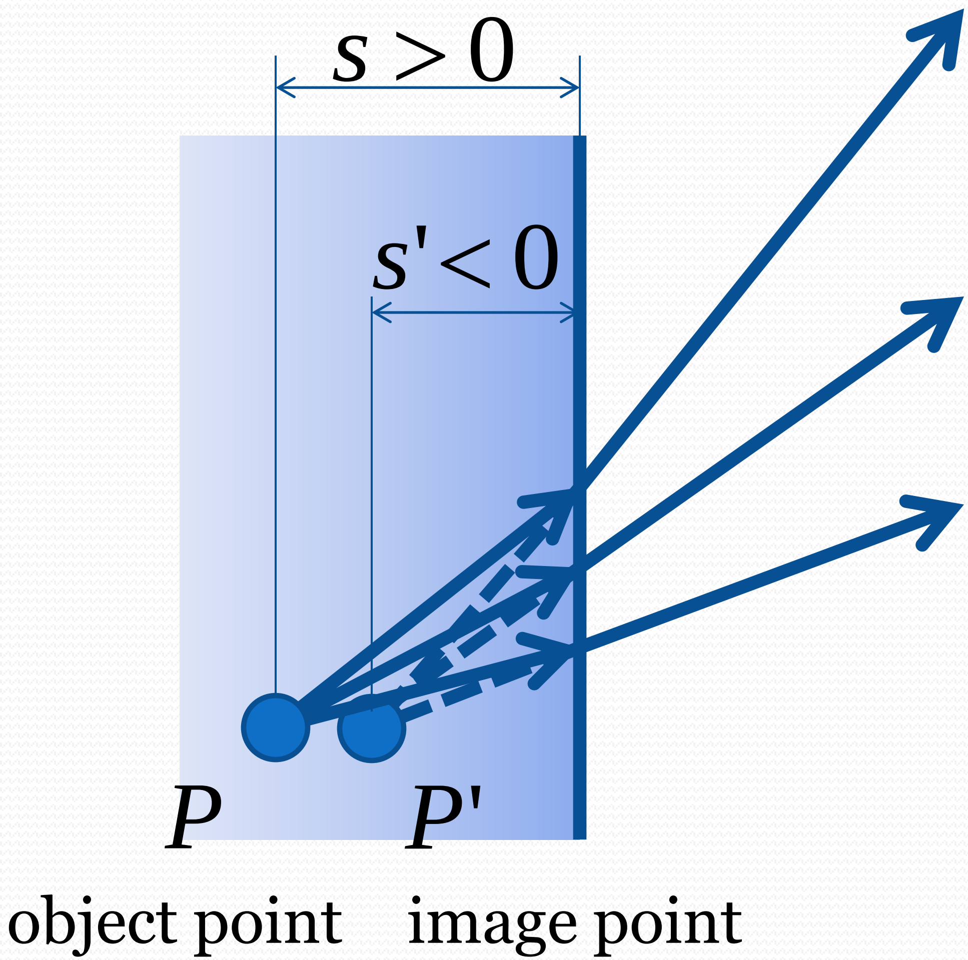
when the object is on the same side as the incoming light, $s>0$, otherwise, $s<0$

**Image distance:**

when the image is on the same side as the outgoing light, $s'>0$, otherwise, $s'<0$

mirror

# Sign Rules for Distances




**Object distance:**

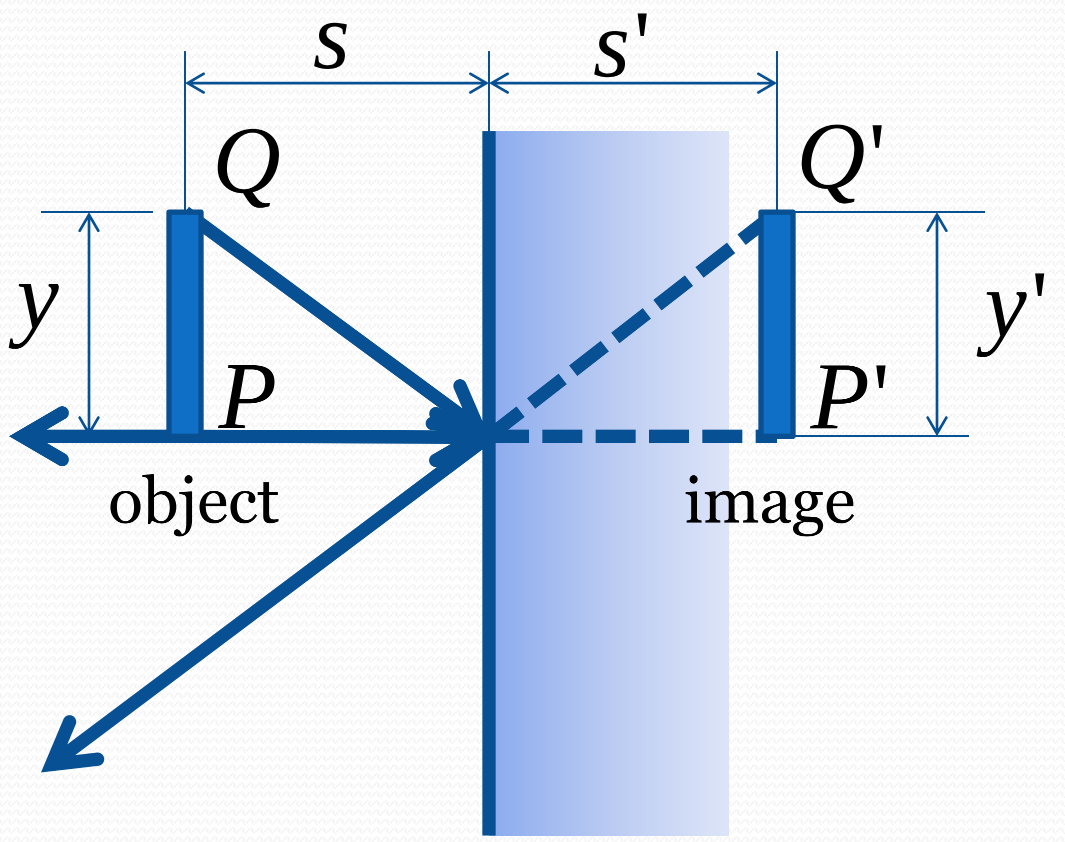
when the object is on the same side as the incoming light, $s>0$, otherwise, $s<0$

**Image distance:**

when the image is on the same side as the outgoing light, $s'>0$, otherwise, $s'<0$

refracting interface

# Lateral Magnification




$$m = \frac{y'}{y}$$

mirror:

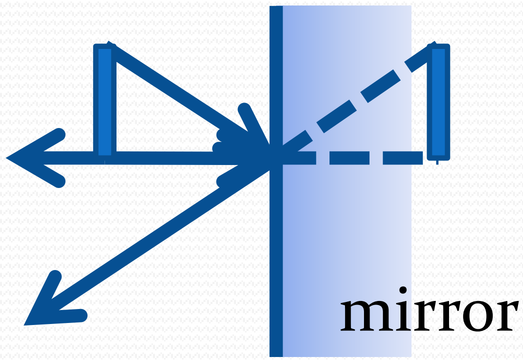
$$s = -s'$$

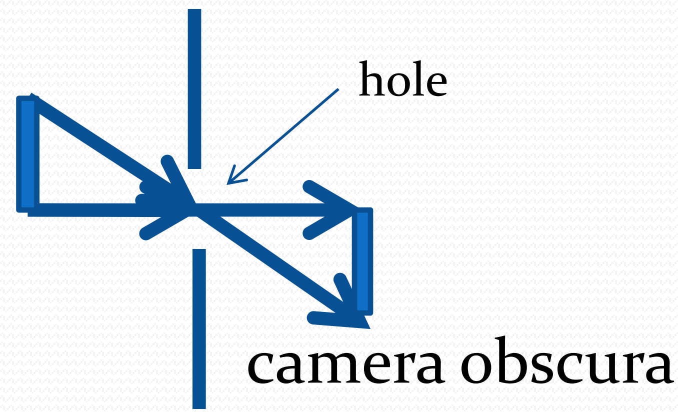
$$m = 1$$

need at least two points ($P$,$Q$) to figure it out

# Inverted and Reversed Images

- Image can be **erect** (right side up) or **inverted** (upside down)



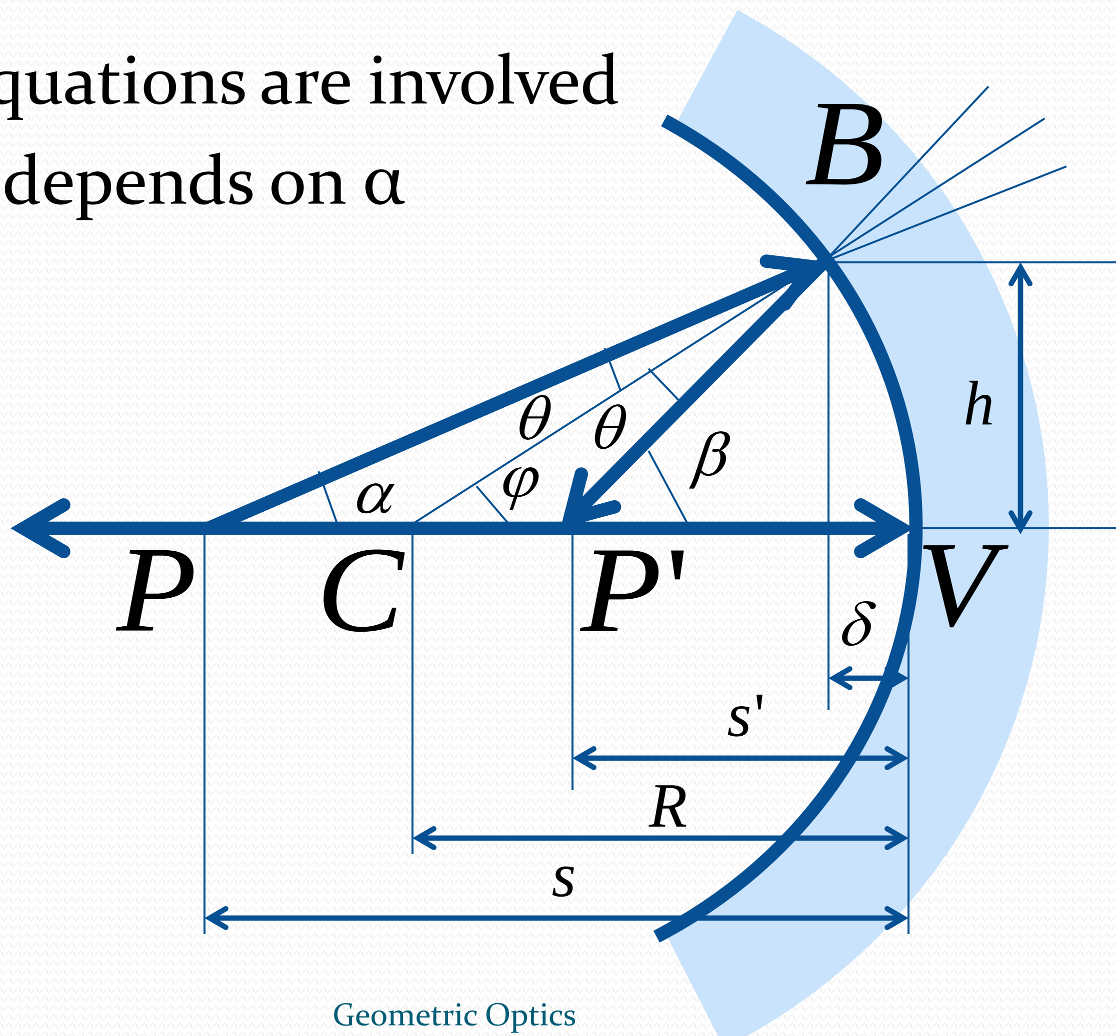




- Image can be **reversed** ("mirror-image" – left hand looks like right and vice versa)

a plain mirror image is **virtual**, **erect**, and **reversed**

# Reflection in a Sphere

- Equations are involved
- *s*' depends on α




$$\varphi = \alpha + \theta$$

$$\beta = \varphi + \theta$$

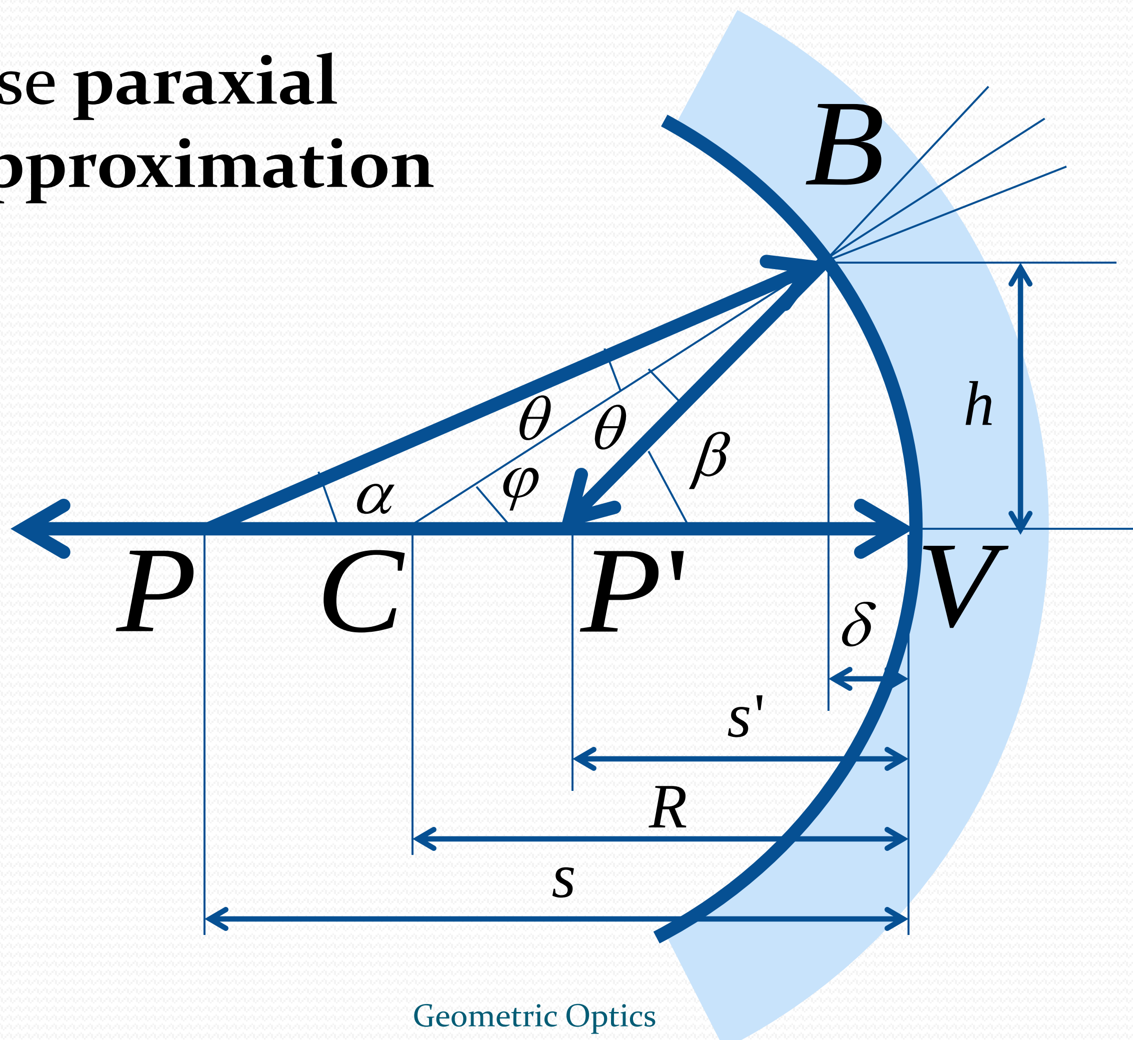
$$\tan\alpha = \frac{h}{s-\delta}$$

$$\tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

# Reflection in a Sphere

- Use **paraxial approximation**




$$\varphi = \alpha + \theta$$

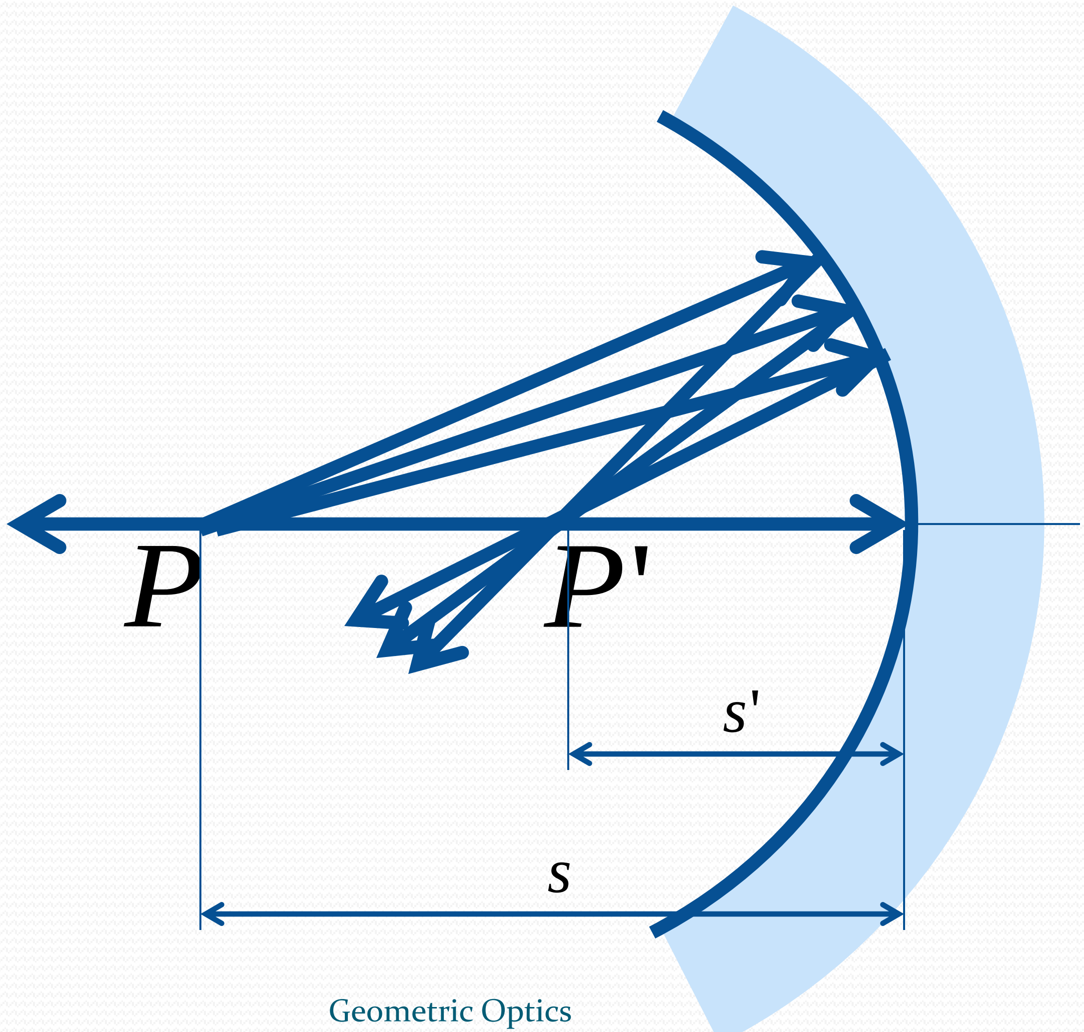
$$\beta = \varphi + \theta$$

$$\cancel{\tan}\,\alpha = \frac{h}{s - \cancel{\delta}}$$

$$\cancel{\tan}\,\beta = \frac{h}{s' - \cancel{\delta}}$$

$$\cancel{\tan}\,\phi = \frac{h}{R - \cancel{\delta}}$$

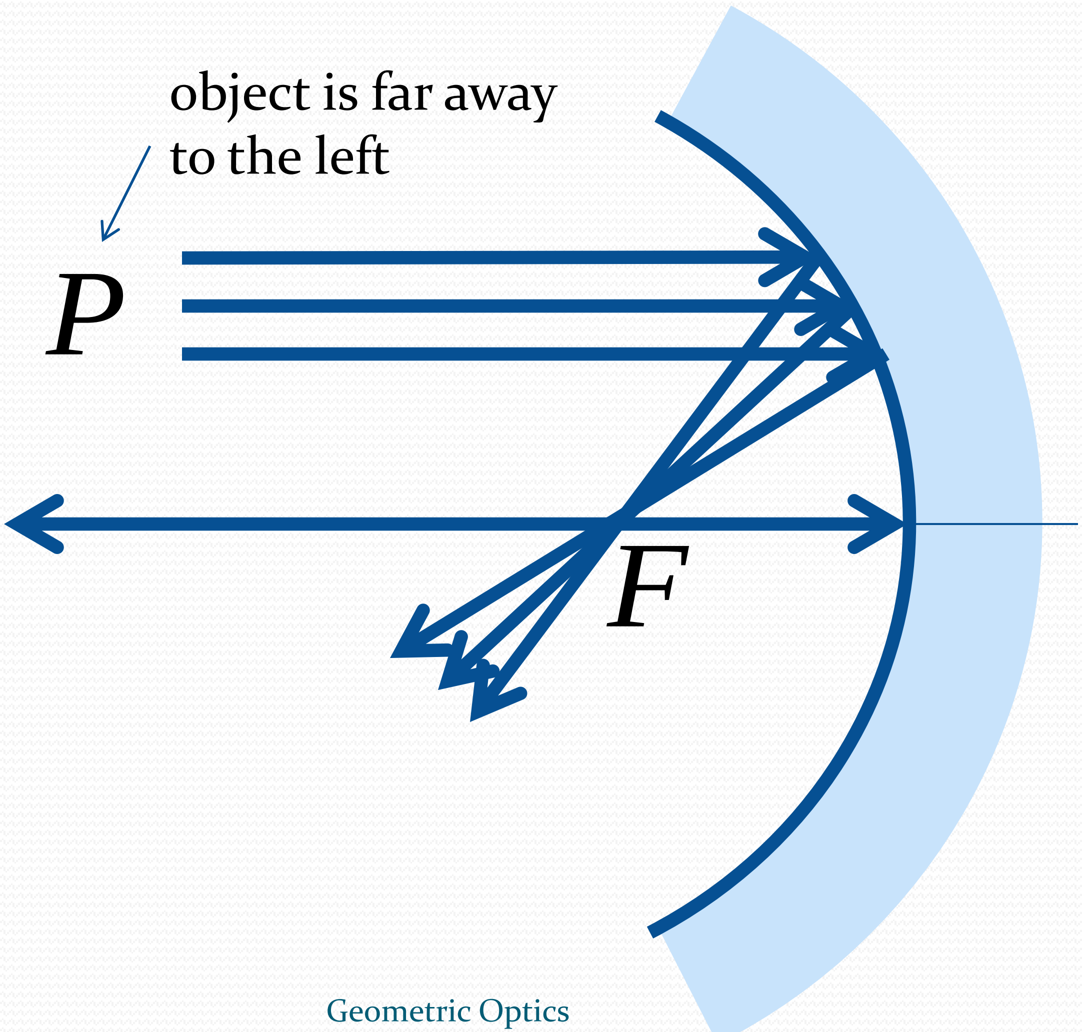
# Spherical Mirror Reflection




$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

no α dependence!

# Focal Point

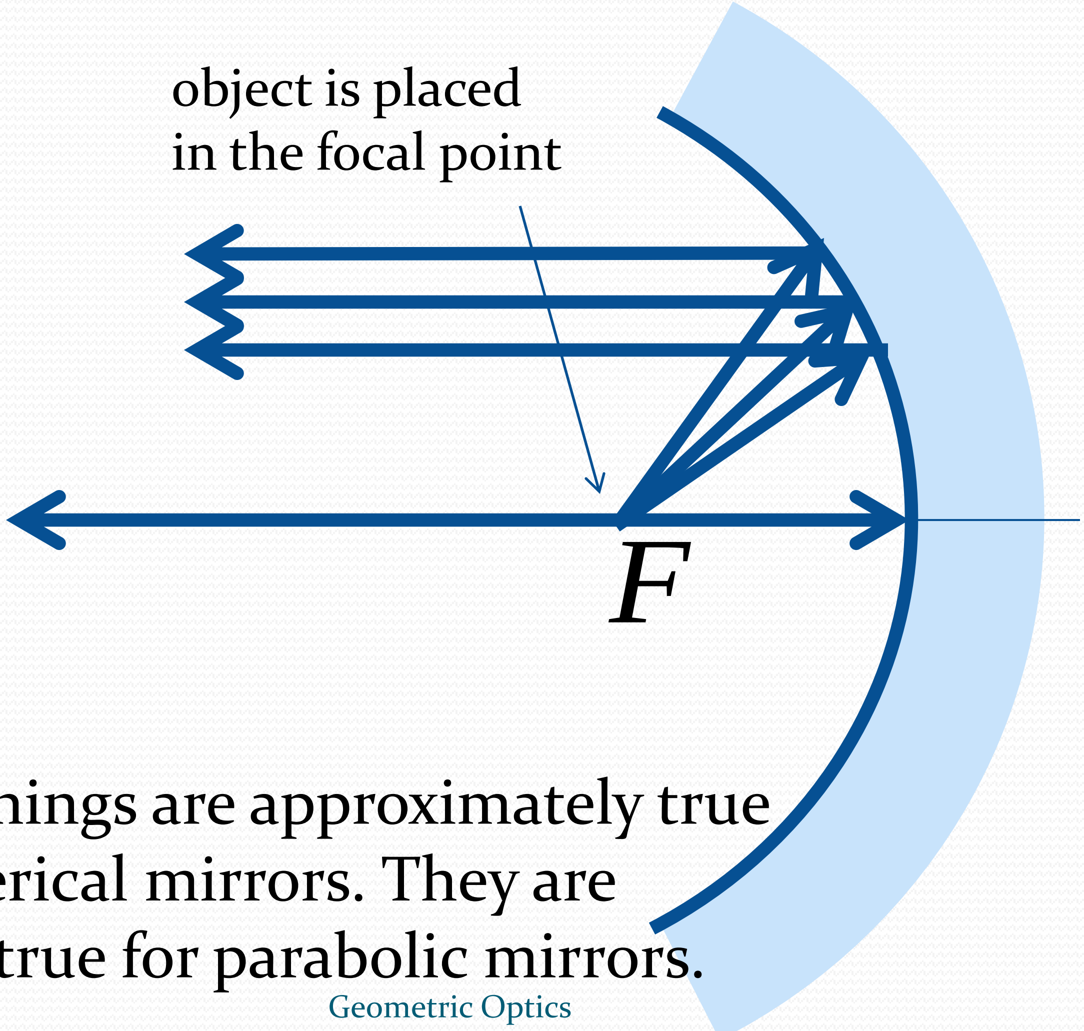



$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R}$$

$$s' = \frac{R}{2} = f$$

**focal length**

# Focal Point



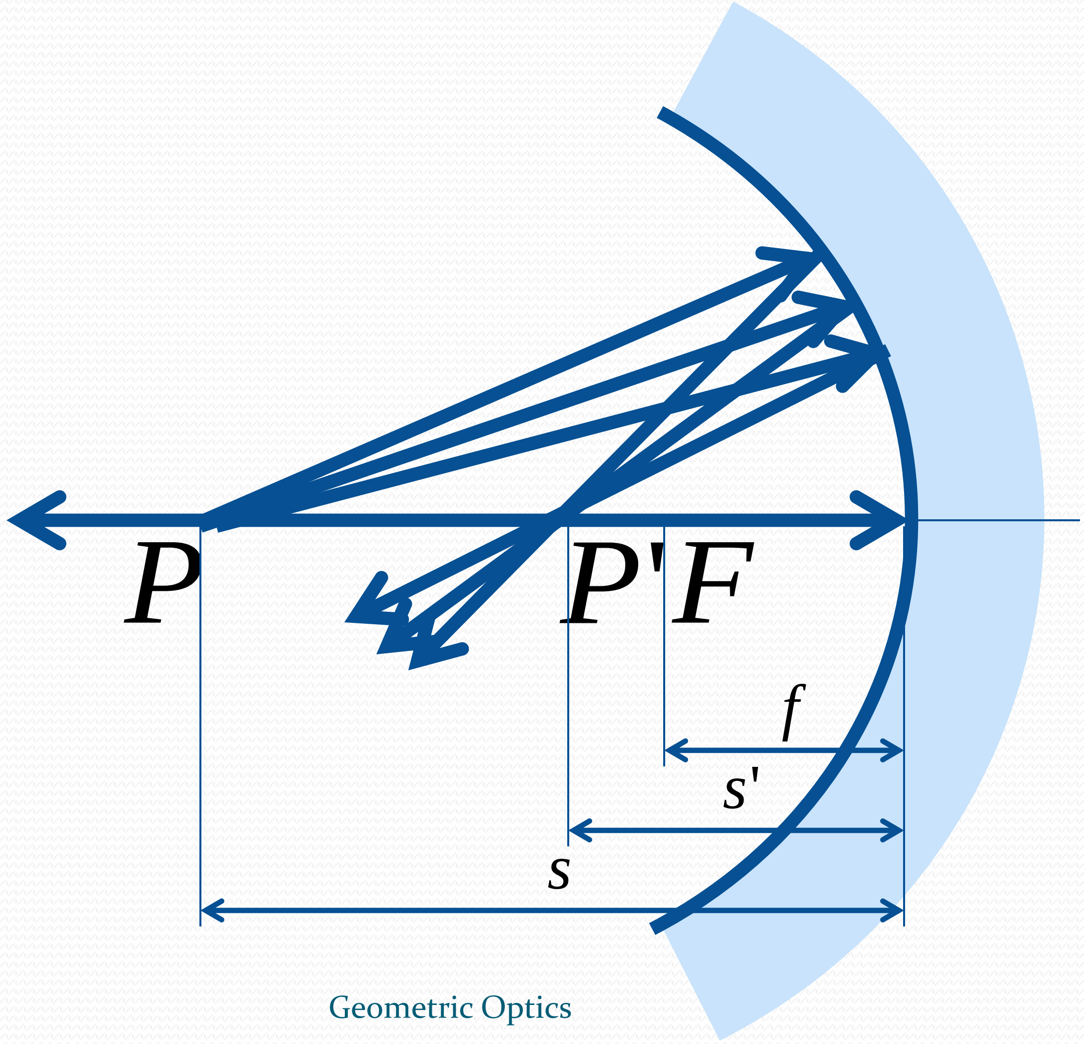

$$\frac{1}{s}+\frac{1}{\infty}=\frac{2}{R}$$

$$s=\frac{R}{2}=f$$

works in both ways

These things are approximately true for spherical mirrors. They are exactly true for parabolic mirrors.

# Spherical Mirror: *s>f*

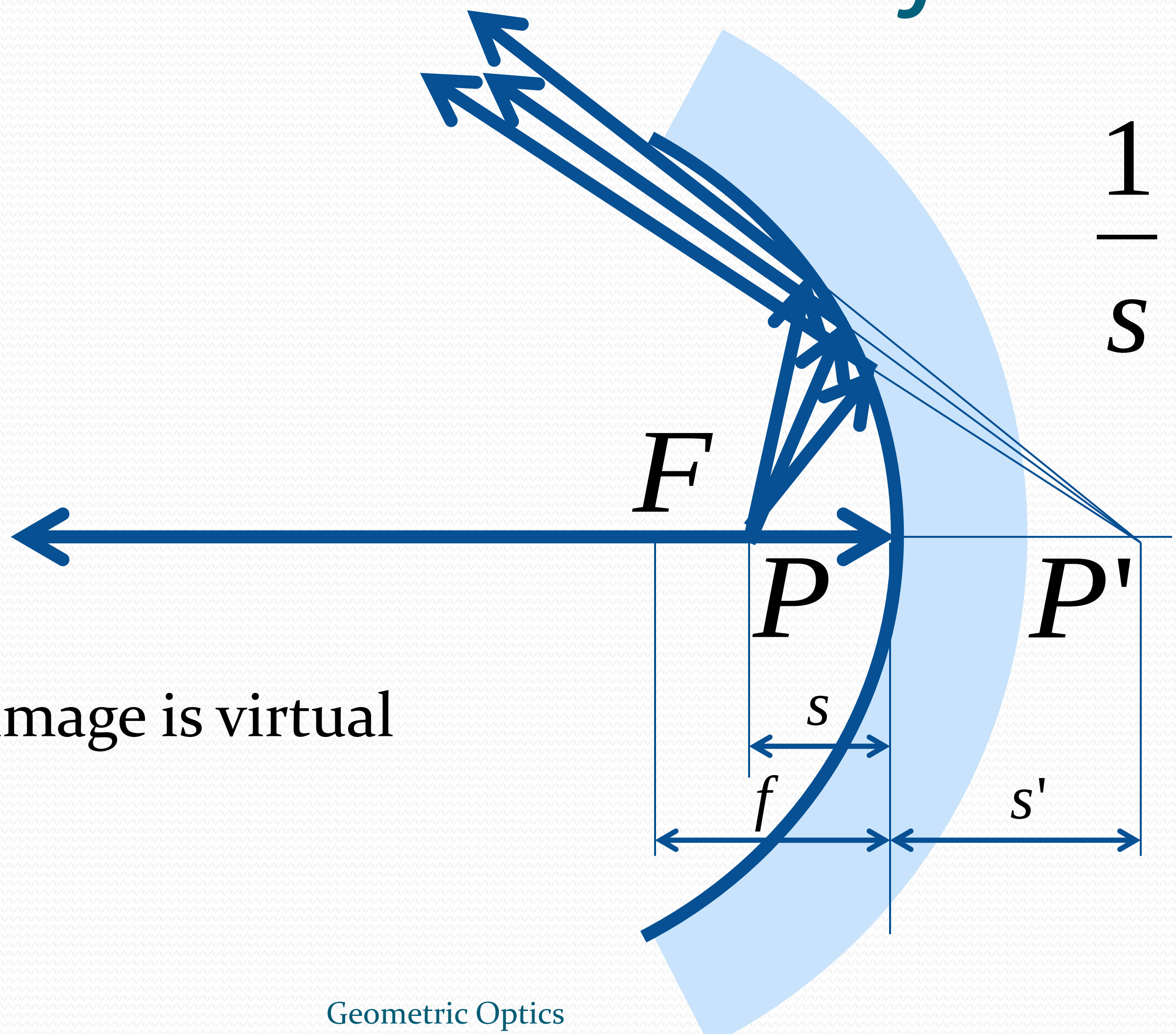



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$s>f$$

$$s'>0$$

# Spherical Mirror: *s<f*



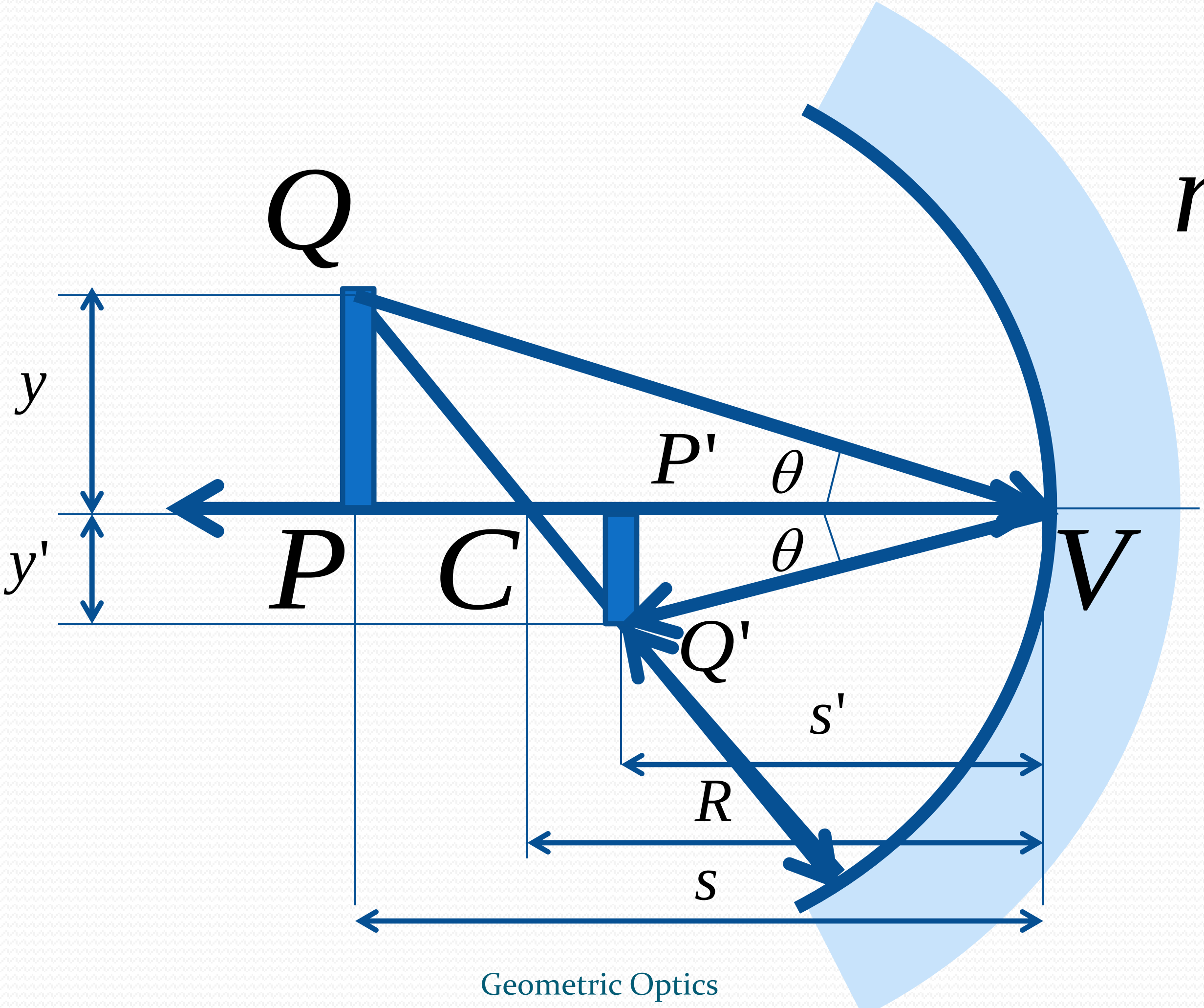

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

image is virtual

$$s<f$$

$$s'<0$$

# Spherical Mirror Magnification



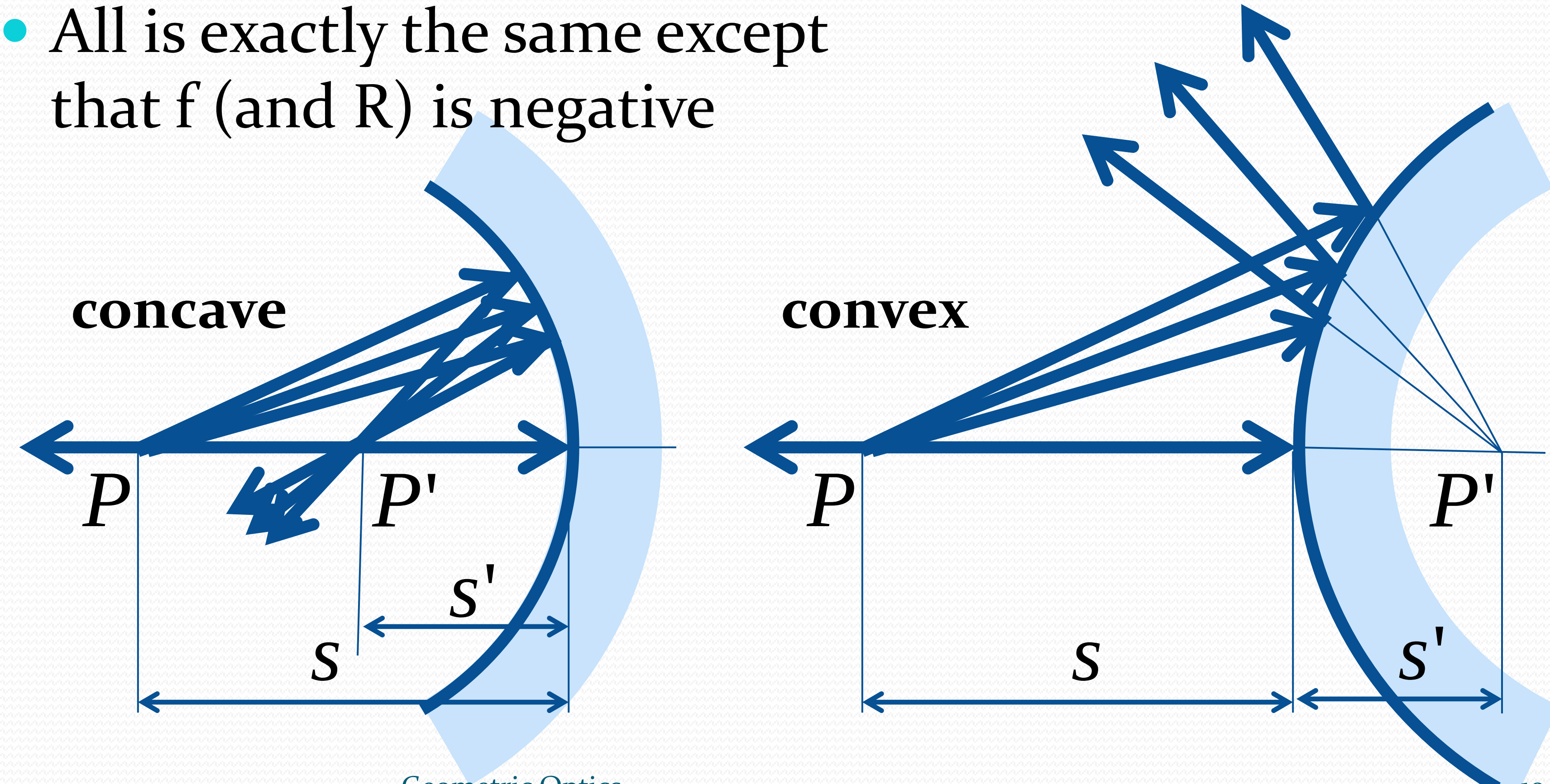

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

because triangles PQV and P'Q'V are similar

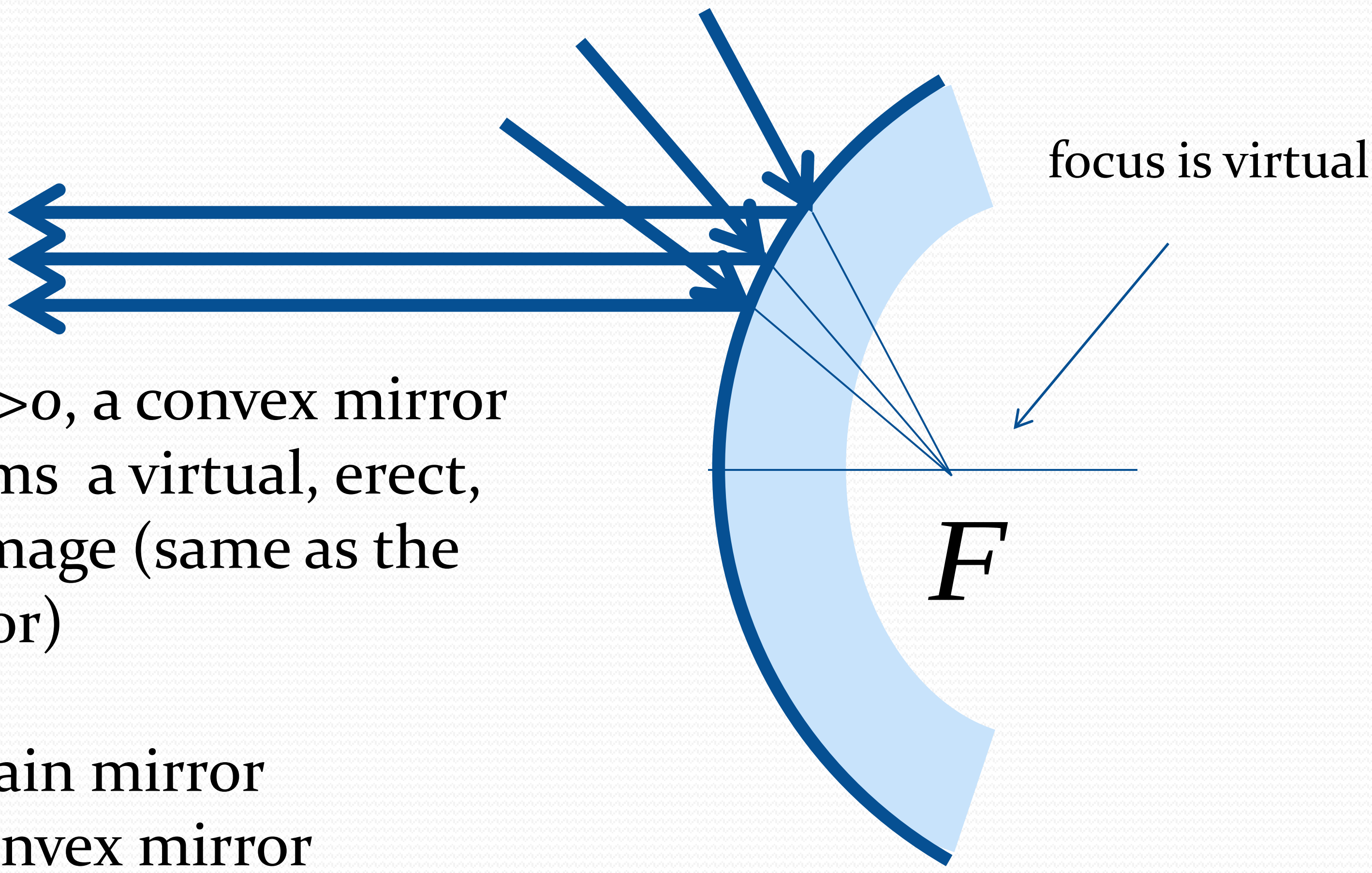
*s'*>0: image is real and inverted ($m<0$)
*s'*<0: image is virtual and erect ($m>0$)

# Convex Spherical Mirror

- **convex** = curving out
- All is exactly the same except that f (and R) is negative



# Focal Point of Convex Mirror

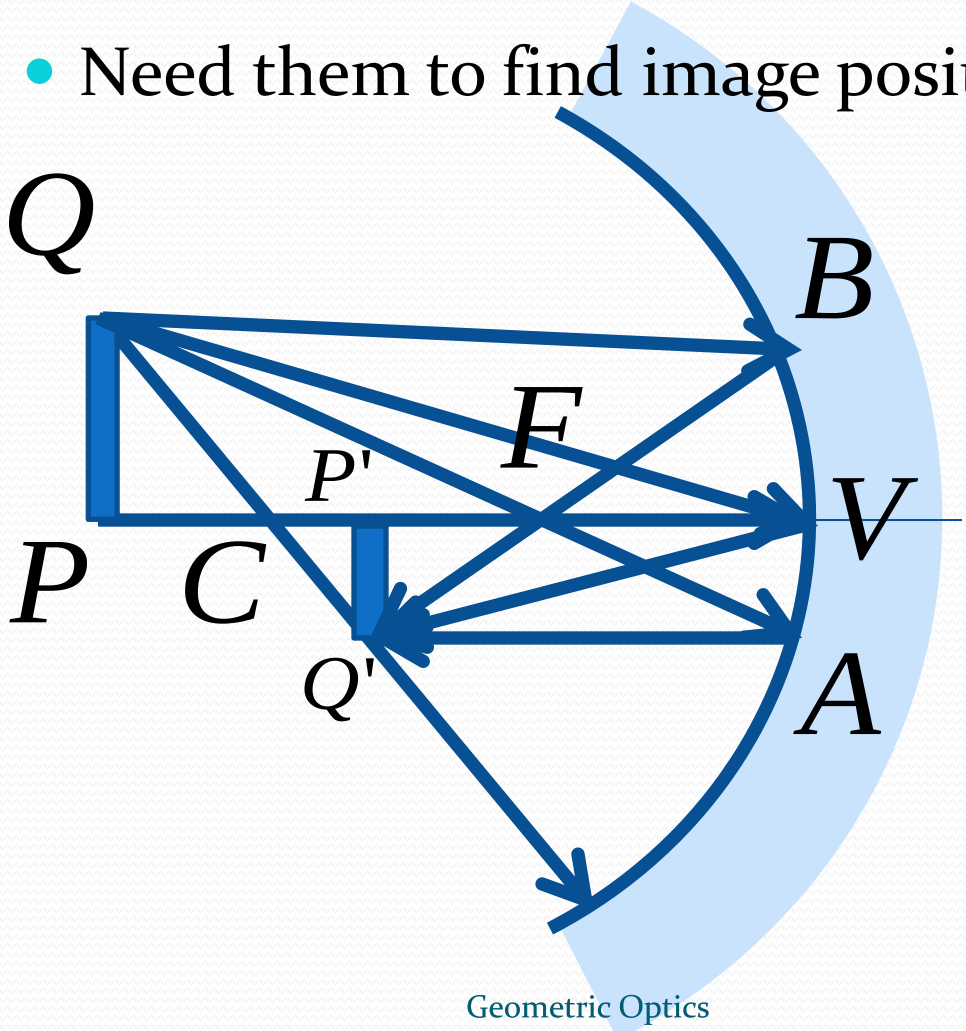



provided $s>0$, a convex mirror always forms a virtual, erect, reversed image (same as the plain mirror)

$m=1$ for plain mirror
$m<1$ for convex mirror

# Principal rays

- Need them to find image position and magnification

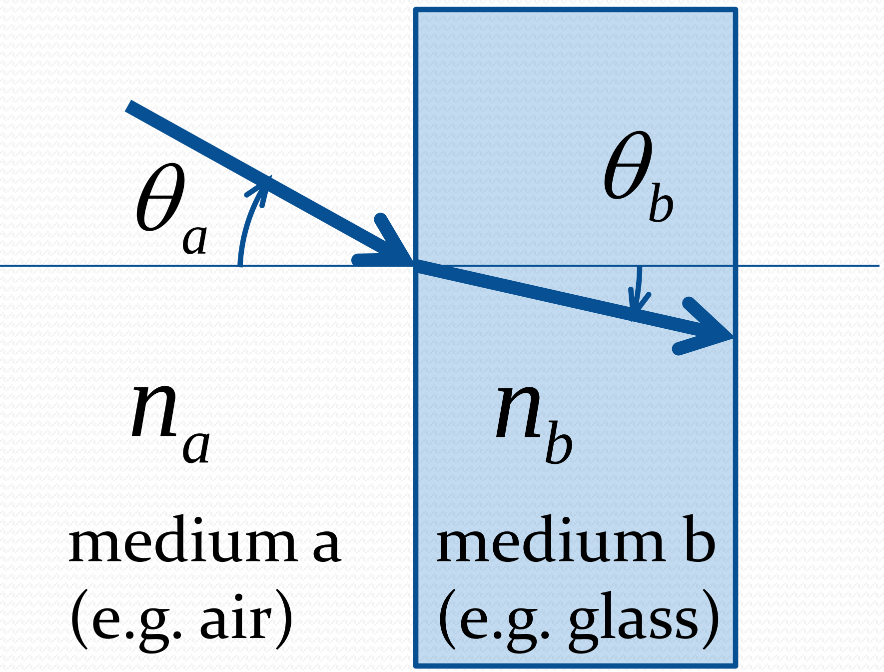



- *QBQ'*: ray parallel to the axis reflects through focal point
- *QAQ'*: ray through focal point reflects parallel to the axis
- *QCQ'*: ray through the center reflects back
- *QVQ'*: ray to the vertex forms equal angles with the axis

this construction neglects aberrations

# Snell's law

- This is the basic law of refraction




angle of incidence not equal to angle of refraction!

$$n_a \sin\theta_a = n_b \sin\theta_b$$

*n*: index of refraction
what is this?

# Index of Refraction

- = ratio of the speed of light in the material to that in vacuum

$$n = \frac{c}{v}$$

$n>1$: light travels slower in the material than in vacuum

What changes when the light passes from one medium to another?

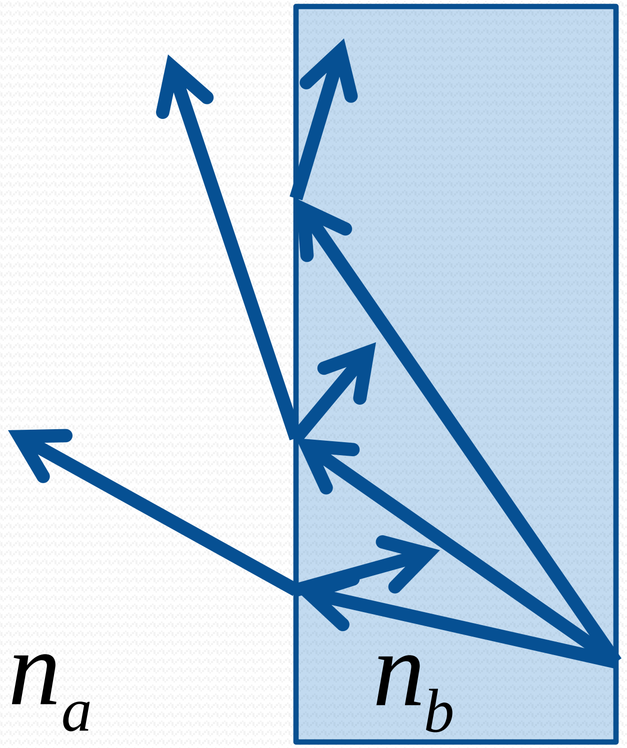
* Frequency ? **No**, it would imply creating/destroying waves

* Speed? **Yes**, because the media have different $n$

* Wavelength? **Yes**, because $\lambda = v/f$

# Total Internal Reflection

- Snell's law may give $\sin\theta > 1$ – what does it mean?




- There are always two rays: reflected and refracted
- At some angle, the refracted ray disappears

$$n_a = n_b \sin\theta_C$$

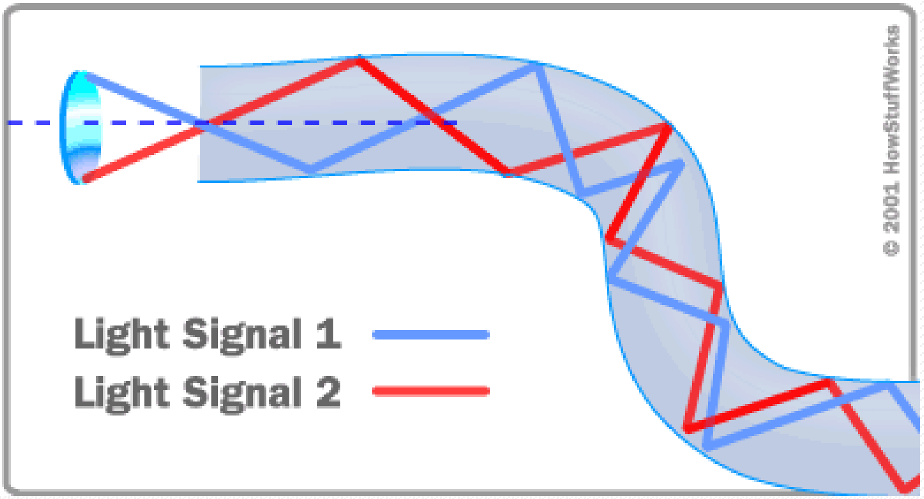
critical angle

can only happen if $n_a < n_b$

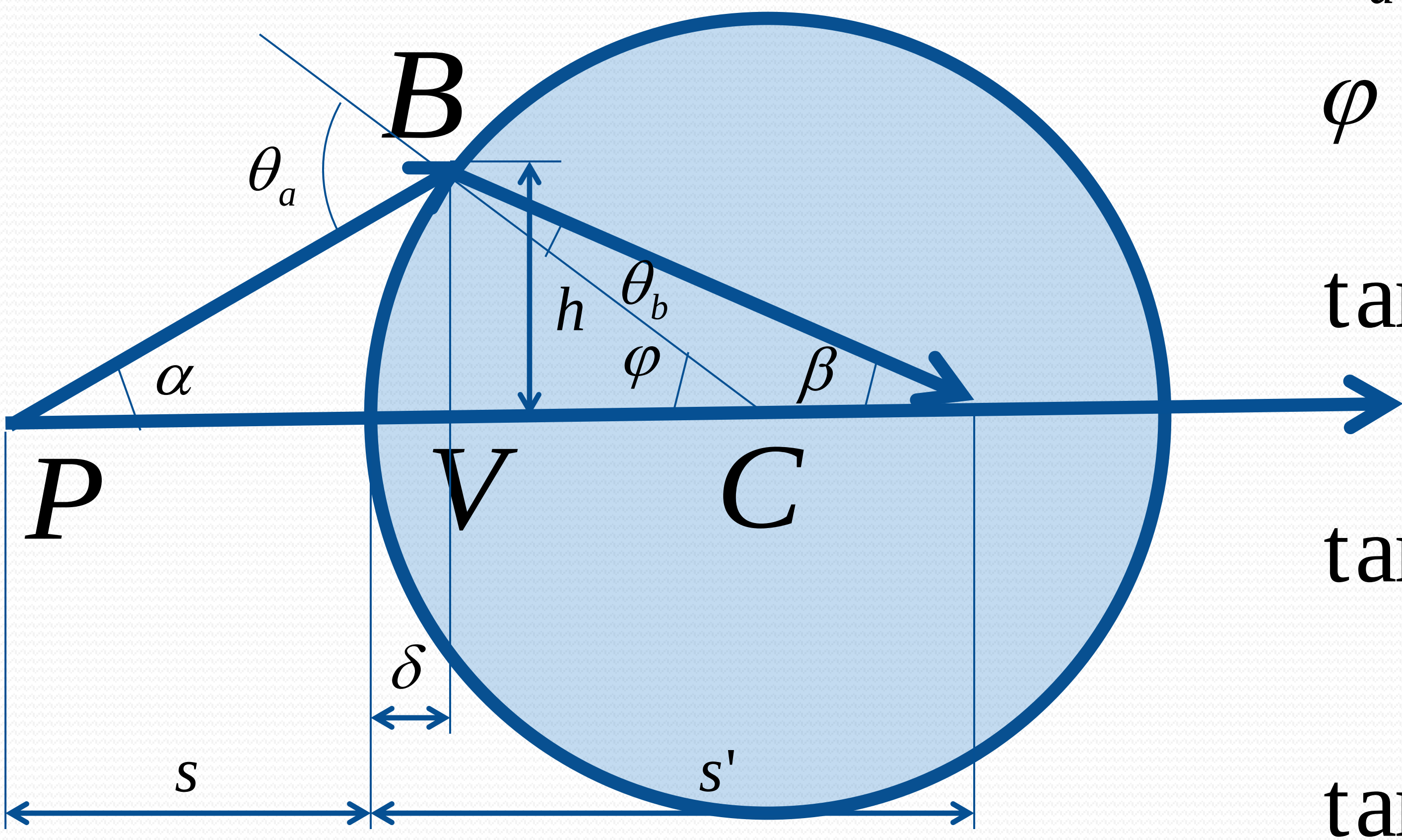
# Fiber Optics

- Light can be transmitted along a fiber with almost no loss due to total internal reflection
  - Due to impurity of glass, the signal eventually degrades (typical rates are ~50%/km)

Widely used in communications – much higher frequency than for regular wires, therefore can transmit much more data



# Refraction at a Sphere




$$\theta_a = \alpha + \varphi$$

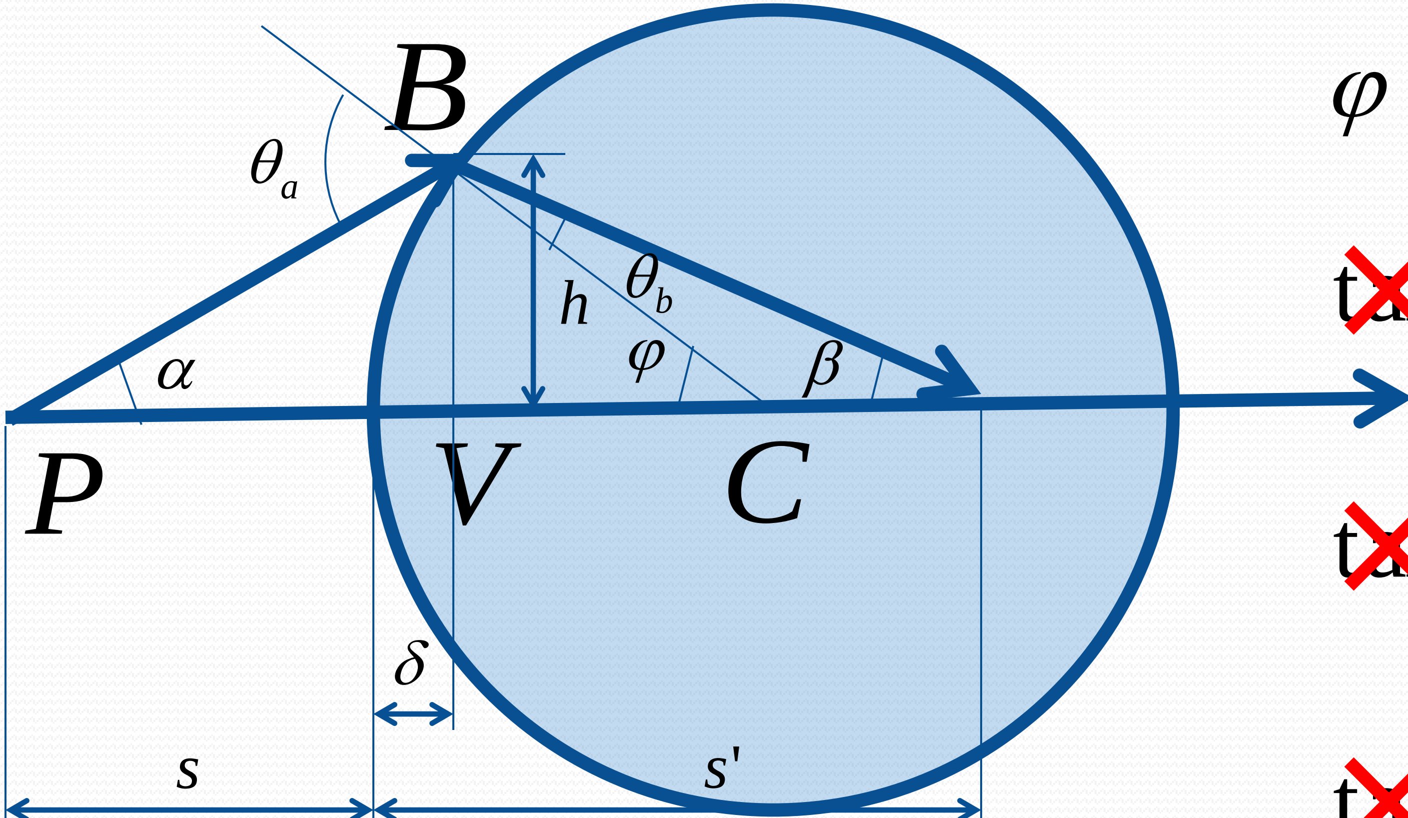
$$\varphi = \beta + \theta_b$$

$$\tan\alpha = \frac{h}{s+\delta}$$

$$\tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

# Refraction at a Sphere




$$\theta_a = \alpha + \varphi$$

$$\varphi = \beta + \theta_b$$

$$\tan\alpha = \frac{h}{s+\delta}$$

$$\tan\beta = \frac{h}{s'-\delta}$$

$$\tan\phi = \frac{h}{R-\delta}$$

# Refraction at a Sphere

- Use Snell's law $n_a \sin\theta_a = n_b \sin\theta_b$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$\theta_a = \alpha + \varphi$$

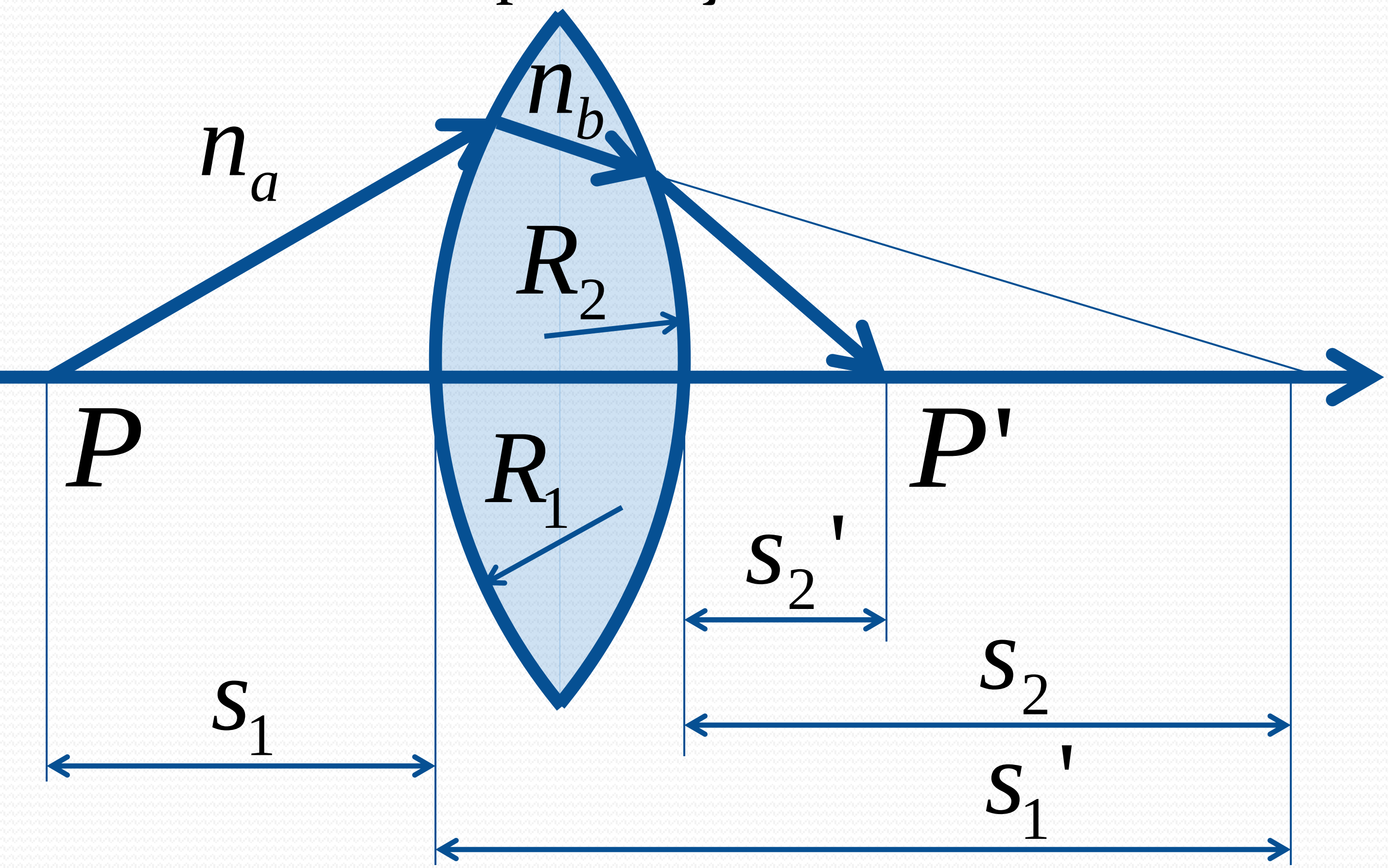
$$\varphi - \beta = \theta_b$$

$$n_a(\alpha + \varphi) = n_b(\varphi - \beta)$$

magnification: $m = -\dfrac{n_a s'}{n_b s}$

# Thin Lens

- Lens = an optical system with two refracting surfaces




**Thin lens:**

$$s_2 \approx -s_1'$$

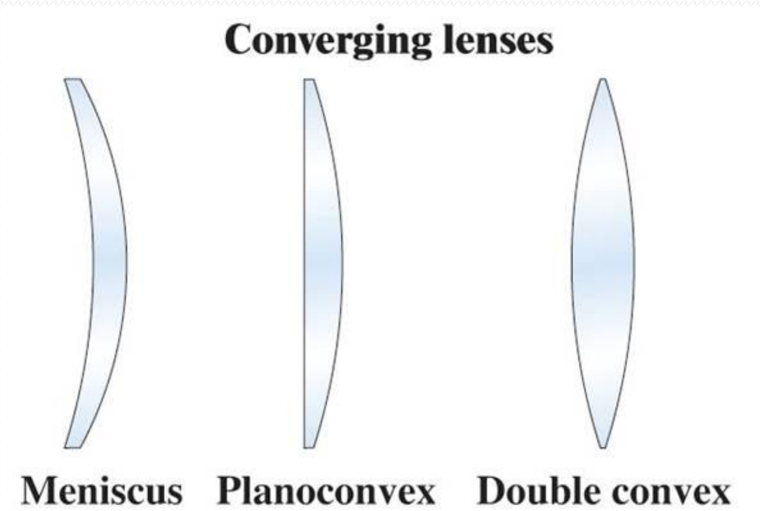
# Thin Lens Equation

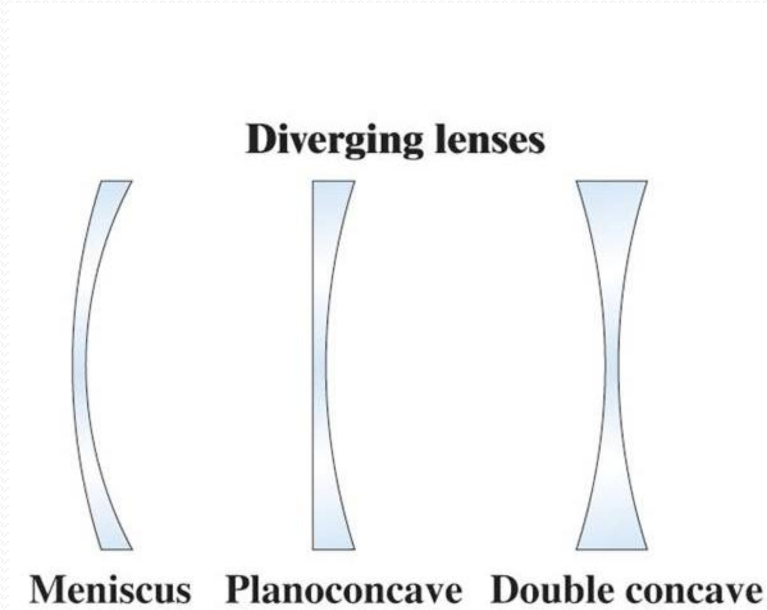
$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}, \quad \frac{n_b}{s_2} + \frac{n_a}{s_2'} = \frac{n_a - n_b}{R_2}$$

$$\text{assumptions: } n_a = 1, \quad n_b = n, \quad s_2 = -s_1'$$

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

# Lens Types




$R_1 < R_2 < 0$

$R_1 = \infty$, $R_2 < 0$

$R_2 < 0 < R_1$

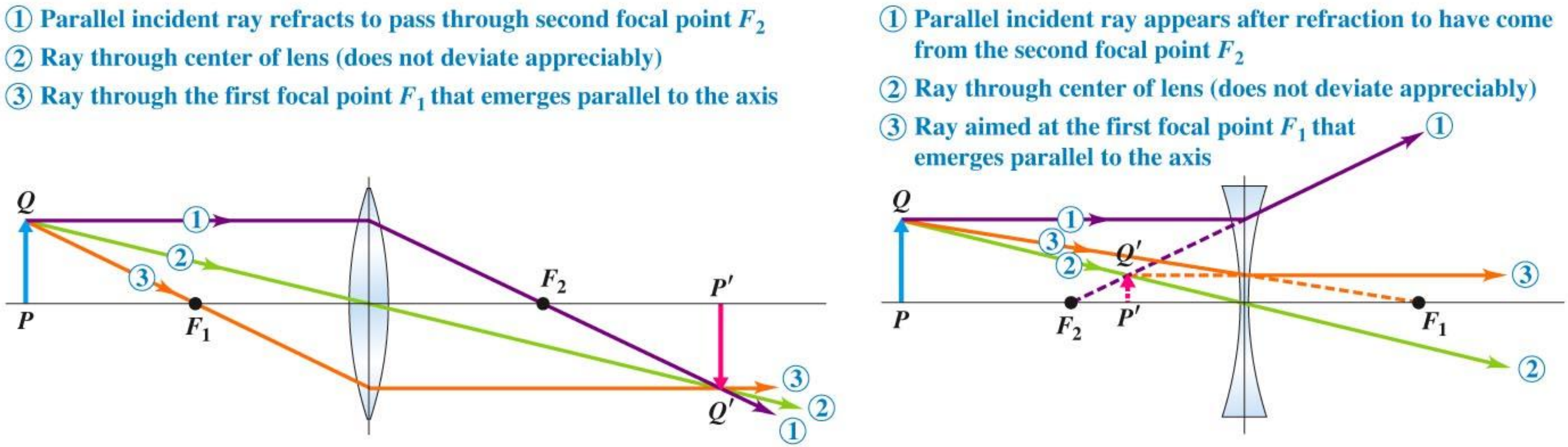
$R_1 > R_2 > 0$

$R_1 = \infty$, $R_2 > 0$

$R_1 < 0 < R_2$

# Graphical Methods for Lenses

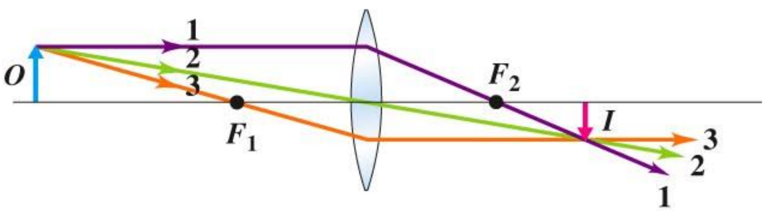
- Be careful with the focal length sign!



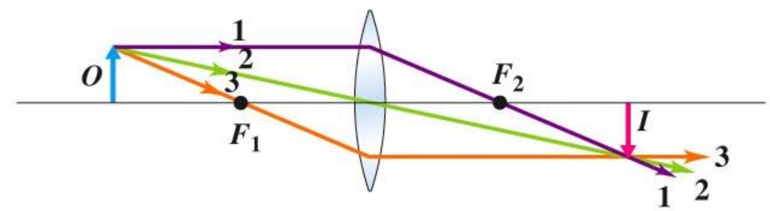

(a) Converging lens

(b) Diverging lens

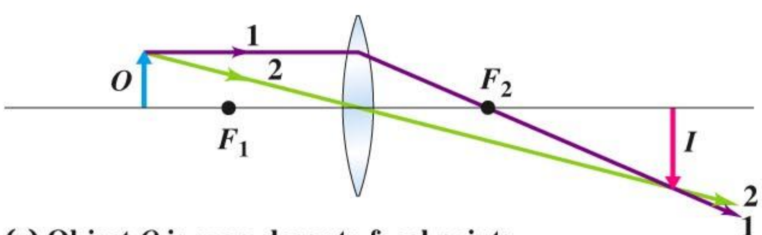
# Graphical Methods for Lenses



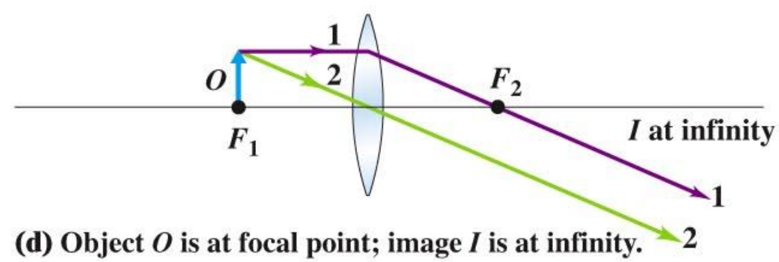

**(a)** Object *O* is outside focal point; image *I* is real.



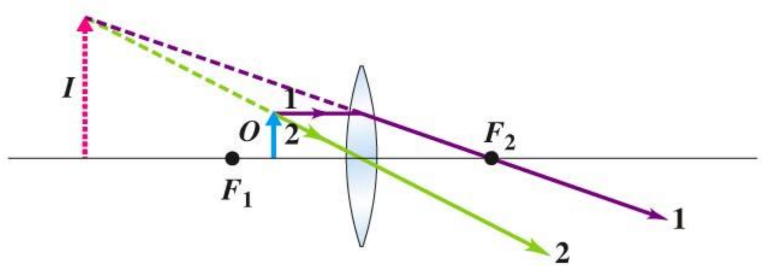

**(b)** Object *O* is closer to focal point; image *I* is real and farther away.



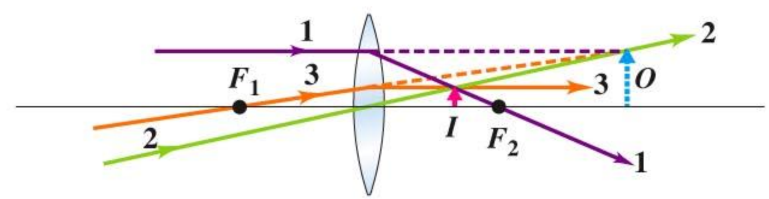

**(c)** Object *O* is even closer to focal point; image *I* is real and even farther away.




**(d)** Object *O* is at focal point; image *I* is at infinity.




**(e)** Object *O* is inside focal point; image *I* is virtual and larger than object.




**(f)** A virtual object *O* (light rays are *converging* on lens)