Chapter 27

Early Quantum Theory and Models of the Atom

Electron

There is something ("cathode rays") which is emitted by the cathode and causes glowing

Unlike light, these rays are deflected by electric and magnetic fields → they are made of charged particles

Originally thought to be molecules or ions, but *e/m* measurements came out with value ~10^3 larger than for hydrogen ions

Beta particles discovered later in studies of radioactive materials turned out to be the same stuff

Light: Particles or Waves?

- 17th century: Newton vs Huygens
  - Newton: light is particles! (win)
- 1800: Young, double slit experiment
  - Fresnel: diffraction experiments
- 1861: Maxwell equations
  - light is electromagnetic waves! (win)
- 1900: Planck, blackbody radiation formula
  - light is… **?**

Blackbody Radiation

- Blackbody = a perfect absorber
  - glass – a bad absorber (for visible light, at least)
  - metal – a bad absorber (it reflects the light)
  - soot – almost perfect absorber
  - a good model of blackbody: a hole in a wall

- Perfect blackbody: all wavelengths are absorbed, nothing is reflected or goes through
- Perfect absorber: been heated, blackbody emits radiation of all wavelengths

Blackbody Radiation

- Experimental fact (Wien's law): blackbody radiation spectrum peaks at $\lambda_{max}=b/T$, where $b=2.90\times10^{-3}$ m·K
- Classical calculation (Rayleigh and Jeans , assuming that radiation is due to atomic oscillations): $I \sim 1/\lambda^4$
  - good for high λ, diverges at low λ (ultraviolet catastrophe)

Planck's Quantum Hypothesis

- Planck proved that everything works fine if one assumes that only certain energies of oscillations are allowed

light comes in pieces!

$$E = nhf$$

$$n = 1, 2, 3\ldots$$

$$I \sim \frac{1}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1 \right)}$$

Planck's constant:

$$h = 6.62 \times 10^{-34}\,\mathrm{J \cdot s} = 4.136 \times 10^{-15}\,\mathrm{eV \cdot s}$$

Photoelectric Effect: Classical View

Classical wave theory of light: energy of light is proportional to intensity <*S*>

$$\langle S \rangle = \langle u \rangle c$$

To kick out an electron, we would need to concentrate light energy in some volume τ

$$\frac{\langle S \rangle}{c}\tau = \frac{1}{2} m v_{\max}^2 = eV_0$$

Max kinetic energy is acquired by the electron if it absorbs it all

$$V_0 \sim \langle S \rangle \quad ??$$

Photoelectric Effect: Experiment

Stopping potential **does not** depend on intensity

Stopping potential **does** depend on light frequency

$$eV_0 = hf - \phi$$

work function

typical values for ϕ: 4—5 eV

Planck's constant (the same as for blackbody radiation!)

Photoelectric Effect: Interpretation

- Einstein: light consists of small particles (photons), each of them carrying energy $E=hf=hc/\lambda$
- When light hits the metal, electron gets either all of the photon energy or none

photons do not have mass

photons always travel with speed of light c

photons carry momentum $p=E/c=h/\lambda$

Atom: What Does It Look Like?

- From X-ray diffraction we know that crystals have lattice constant ~10^{-10} m, so that must be the order of the atom size
- Can't see the atom by eye: atoms are much smaller than λ of visible light!
- Can't do much with X-rays either – how will you make a lens for X-rays?
- So what should we do? **Scattering**.

Rutherford Scattering

- α-particles are massive (~7300 m_e), so they should ignore electrons inside the atom
- what was expected: relatively uniform angular distribution, depending on atom’s density
- what was observed: most α-particles went through undisturbed, but some reflected at very large angles (up to 180°)

Planetary Atom Model

- Problems:
  - according to EM theory, bending charge radiates EM waves, so electrons would lose their energy and fall onto the nucleus real quick
  - atoms show sharp spectral lines, like electrons can’t radiate an arbitrary amount of energy

Bohr’s Atom Model

- Coulomb’s law and energy conservation are OK
- Electrons can only exist at certain orbits (=energy levels) and do so without radiating energy
- Emission/absorption occurs when electron makes a transition between two stationary states: $hf=E_i-E_f$
- The angular momentum is quantized

Angular Momentum Quantization

- Bohr’s hypothesis:

$$mvr = n\hbar$$

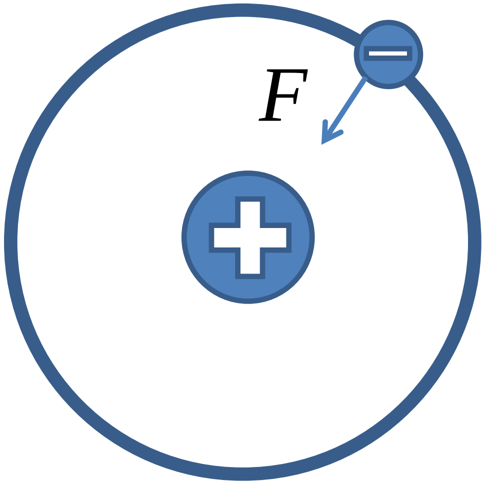
electron’s mass

electron’s speed

electron’s orbit radius

1,2,3, …

some constant



Coulomb’s law:

$$F = k\frac{e^2}{r^2}$$

Centripetal force:

$$F = m\frac{v^2}{r}$$

$$r = \frac{n^2\hbar^2}{kme^2}$$

Bohr Orbits

$$r = \frac{n^2 \hbar^2}{kme^2}$$

What is $\hbar$?
For the first orbit (n=1),
assuming $r \sim 10^{-10}$ m,
one gets ~1.4×10^{-34} J·s

Correct answer: $\hbar = \frac{h}{2\pi}$ ← **Planck's constant**

Electron Energy Levels

$$r = \frac{n^2\hbar^2}{kme^2} \qquad v = \frac{n\hbar}{mr} = \frac{ke^2}{n\hbar}$$

$$U = -k\frac{e^2}{r} = -\frac{mk^2e^4}{n^2\hbar^2} \qquad K = \frac{mv^2}{2} = \frac{mk^2e^4}{2n^2\hbar^2}$$

$$E_n = K + U = -\frac{mk^2e^4}{2n^2\hbar^2}$$

Electron Energy Levels

$$\frac{mk^2e^4}{2\hbar^2} = 13.6\text{ eV}$$

$$E_n = -\frac{13.6\text{ eV}}{n^2}$$

ground state energy, and also ionization energy

Rydberg constant

emitted photon energy: $E_\gamma = E_n - E_{n'}$

emitted photon wavelength: $\frac{1}{\lambda} = \frac{E_\gamma}{hc}$

$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

Emission Lines

Balmer series (*n'*=2)

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Other series: Lyman (*n'*=1, UV); Paschen (*n'*=3), Bracket (*n'*=4), Pfund (*n'*=5 – all IR)

It was discovered first because it (partially) lies in the visible spectrum

It can be seen in the solar spectrum, which confirms presence of hydrogen