Chapter 1

Physics and Measurements
Length, Mass, and Time

- These are the three fundamental quantities mechanics is concerned about.
- All other quantities in mechanics can be expressed in terms of these three:
  - distance: it’s length, too (length of a pole = distance between its ends)
  - width, height, depth: it’s all length
  - frequency: how many times something occurs per unit time (so it’s defined through time)
  - speed: distance traveled per unit time
  - temperature: mechanics doesn’t care 😊
Measurements and Standards

- Physics wants to measure everything
  - measure = assign a number
- Measurements are relative: we compare properties of an object to something we already know
  - a tree is as tall as a ten-storey building
  - the FAST radio telescope is as large as 30 football fields
- To make measurements reproducible, the results should be compared to standards, which must
  - be readily accessible
  - yield the same result anywhere in the Universe
  - not change with time
Time

- Unit: second
- Old definition: \( \left( \frac{1}{24} \right) \left( \frac{1}{60} \right) \left( \frac{1}{60} \right) \) of mean solar day
  - can’t use this definition away from Earth
  - Earth rotation is irregular and slowing down
- Modern definition: 9 192 631 770 \( \times \) period of vibration of radiation from the Cesium-133 atom
  - it’s “indirect” because what’s measured is frequency, not the time interval – but that’s OK (T=1/f)
  - it’s practical (easy to make a clock), works everywhere in the Universe, and is perfectly stable (as far as we know)
Length

• Unit: meter

• Old definitions:
  • fraction of Earth’s perimeter
  • artifact-based – a bar stored in France
  • wavelength of orange-red light emitted by a Krypton-86 lamp

• Modern definition: the distance traveled by light in vacuum during \( \frac{1}{299\,792\,458} \) second
  • speed of light is postulated (and can’t be measured)
  • there is no need to have two separate standards – it follows from a deep relation between space and time revealed by the special theory of relativity
Mass

- Unit: kilogram
- Old definition
  - artifact-based – a special alloy cylinder kept in France
- Modern definition
  - it’s still the same 😞
- What to do? Follow the meter 😊
  - Avogadro project: fix the mass of an atom (Silicon-28)
  - Watt balance: fix Planck’s constant
- No official decision yet
Dimensional Analysis

Two reasons to do it:
  - weed out errors in calculations
  - evaluate dependencies between the quantities

Is the formula for position $x = \frac{1}{2} at^2$ correct?
  - $[a]=L/T^2$  $[t]=T$  $[x]=L$  – correct 😊

If we know acceleration and time, how could position depend on them?
  - if there are no other factors, we can assume that $x = a^n t^m$
  - $L=(L/T^2)^n T^m$  $n=1$;  $m-2n=0$  $\Rightarrow$  $m=2$  $x \sim at^2$
## SI Units

- **Fundamental units:** meter m, kilogram kg, second s
- **Prefixes:** Xm, Xg, Xs, where X is one of SI symbols below:

<table>
<thead>
<tr>
<th>SI PREFIX</th>
<th>SI SYMBOL</th>
<th>SI UNIT CONVERSION FACTOR (STANDARD FORM)</th>
<th>FACTOR (POWER)</th>
<th>FACTOR LANGUAGE</th>
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<tbody>
<tr>
<td>yotta</td>
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Conversion of Units

- Simple conversions:
  - 10 in = 10 in \(\times\) (2.54 cm/1 in) = 25.4 cm
  - 2 cm = 2 cm \(\times\) (1 in/2.54 cm) = 0.787 in

- Complex conversions:
  - 1 m² = 1 m² \(\times\) (100 cm/1 m)² = 10 000 cm²
  - 1 m/s = 1 m/s \(\times\) (1 km/1000 m)/(1 h/3 600 s) = 3.6 km/h
  - 60 mi/h = 60 mi/h \(\times\) (1 609 m/1 mi)/(3 600 s/1 h) = 29 m/s
Scientific Notation

- Scientific notation = a number expressed in form $a \times 10^b$
  - can be done in more than one way
- Normalized scientific notation: $1 \leq a < 10$
  - not defined for zero
  - if $b=0$, the $10^b$ factor is omitted
- Engineering notation: $b$ is a multiple of 3
  - makes it easy to match prefixes
Order of Magnitude Estimates

- Order of magnitude estimate = a power of ten
- Prescription:
  - write the number in normalized scientific notation: $x = a \times 10^b$
  - if $a < \sqrt{10}$, $x \sim 10^b$, otherwise $x \sim 10^{b+1}$
Significant Figures

• When quantities are measured, the results are known within experimental uncertainty

• 15 000 m – not clear
  • should we trust all 5 digits? (probably not)

• $1.5 \times 10^4$ m – better
  • there are just two digits to trust (could be anywhere between 14 500 m and 15 500 m)
  • $1.5 \times 10^4$ m and $1.50 \times 10^4$ m mean different things

• $(1.5 \pm 0.1) \times 10^4$ m – a lot of information
  • in a paper, this would mean that the probability that the actual quantity is between 14 900 m and 15 100 m is 68%
Operations with Approximate Numbers

- When adding/subtracting, keep the smallest number of decimal places
  - $1.40 + 2.41 = 3.81$ (3 s.d. + 3 s.d. -> 3 s.d.)
  - $1.4 + 2.41 = 3.8$ (2 s.d. + 3 s.d. -> 2 s.d.)
  - $1.205 - 1.203 = 2 \times 10^{-3}$ (4 s.d. - 4 s.d. -> 1 s.d.)
  - $12.05 - 1.203 = 10.85$ (4 s.d. - 4 s.d. -> 4 s.d.)

- When multiplying/dividing, keep the smallest number of significant digits
  - $1.40 \times 2.41 = 3.37$ (3 s.d. \times 3 s.d. -> 3 s.d.)
  - $\pi(2.1)^2 = 14$ (s.d. \times 2 s.d. -> 2 s.d.)