Chapter 11

Angular Momentum

reminder

Scalar Product of Two Vectors

- Scalar product = dot product
- It’s defined for two vectors of the same dimension
- Coordinate independent definition:

$$c = \vec{A} \cdot \vec{B} = AB\cos\theta \qquad \vec{A} \cdot \vec{A} = A^2$$

- Coordinate related definition :

$$c = (a_x\vec{i} + a_y\vec{j} + a_z\vec{k})(b_x\vec{i} + b_y\vec{j} + b_z\vec{k})$$

$$= a_x b_x + a_y b_y + a_z b_z$$

Vector Product of Two Vectors

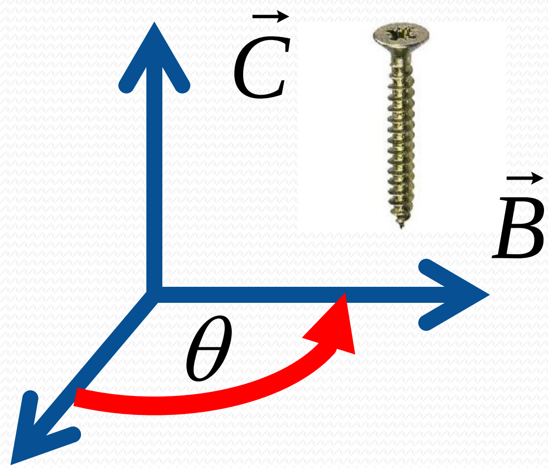
- Coordinate independent definition: vector perpendicular to both ***A*** and ***B***
- Coordinate related definition:

$$\vec{C} = \vec{A} \times \vec{B}$$

$$C = AB\sin\theta$$

$$\vec{A} \times \vec{A} = 0$$

$$\vec{C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

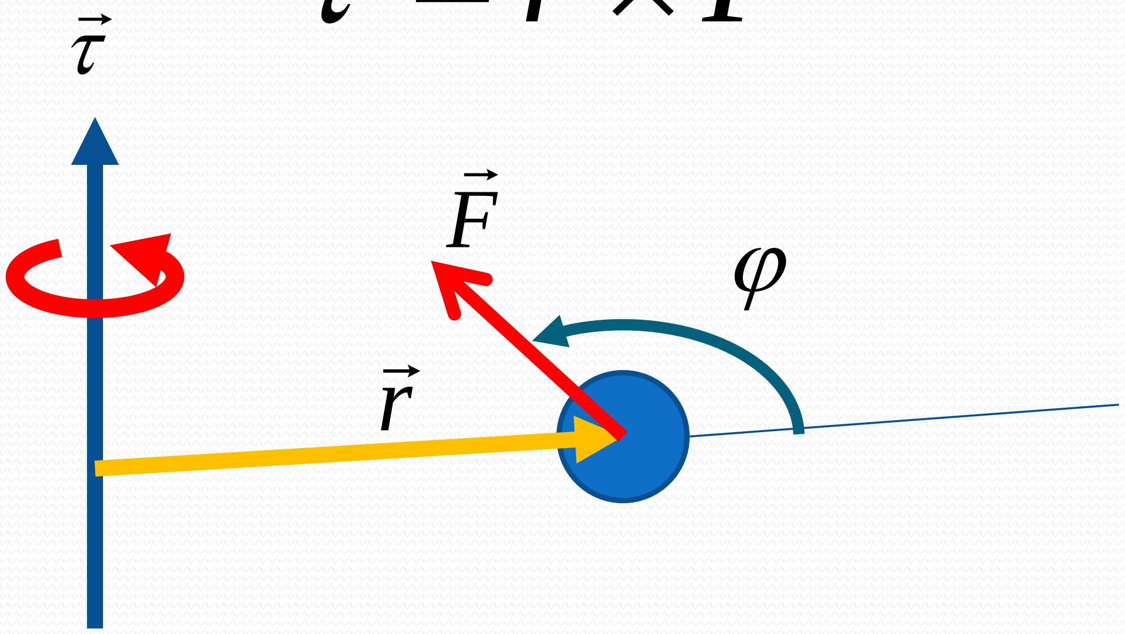


$$= (A_y B_z - A_z B_y)\vec{i} + (A_z B_x - A_x B_z)\vec{j} + (A_x B_y - A_y B_x)\vec{k}$$

Torque as a Vector

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = Fr\sin\varphi$$



Angular Momentum

- Definition: $\vec{L} = \vec{r} \times \vec{p}$

linear momentum

$$\vec{p} = m\vec{v}$$

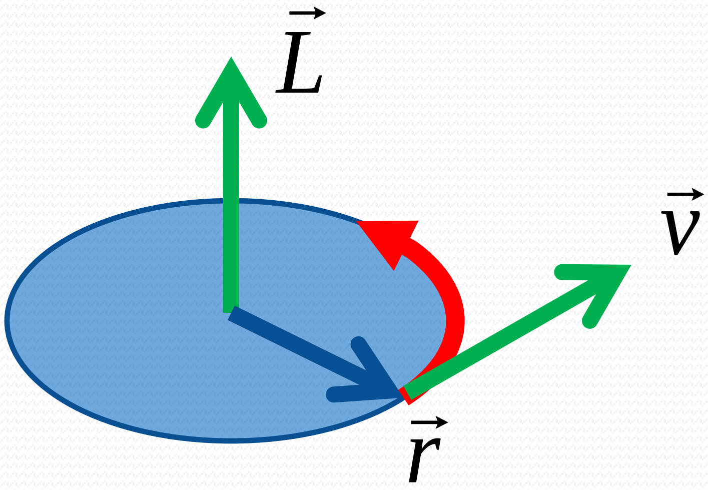
$$\frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times \vec{F} = \vec{\tau}$$

=0

Angular Momentum of a Rotating Rigid Object

$$L_z = \sum_i r_i m_i v_i = \sum_i r_i m_i r_i \omega = \omega \sum_i m_i r_i^2 = I_z \omega$$



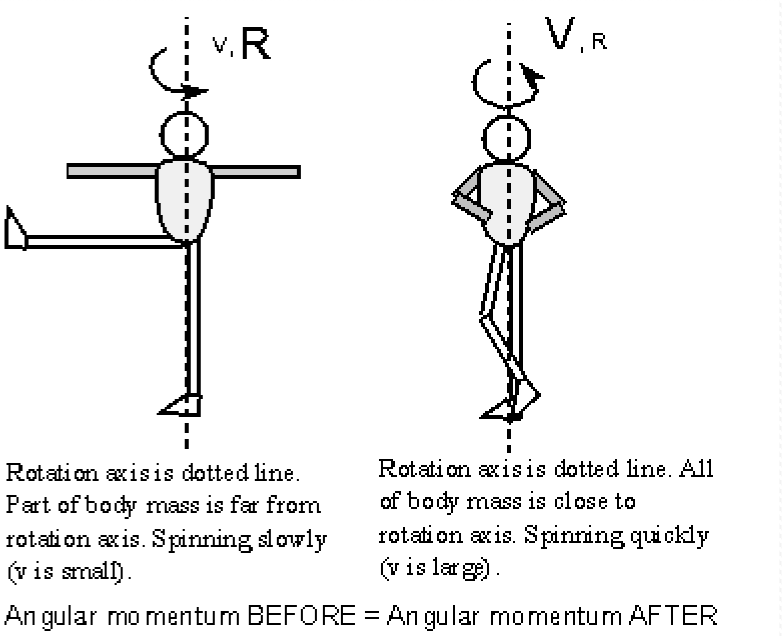
$$\tau = \frac{dL_z}{dt} = I_z \alpha$$

we derived it before!

Careful: in general, ***L***=*I**ω*** is not always valid
(and ***L*** and ***ω*** do not have to be parallel!)

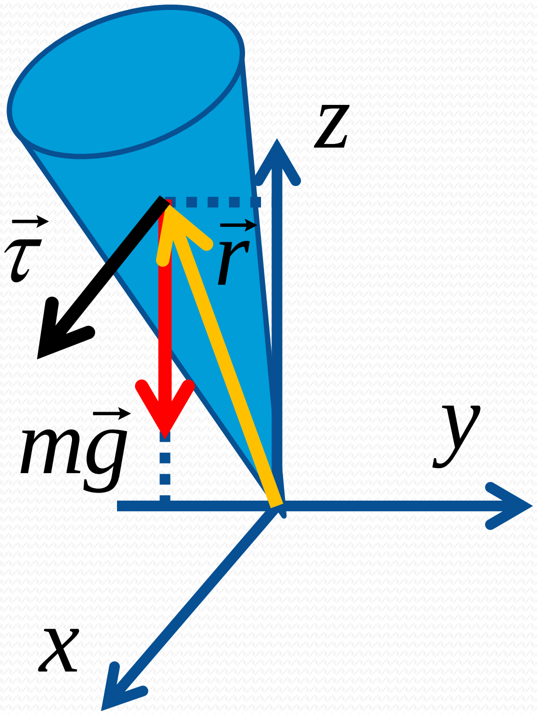
Conservation of Angular Momentum

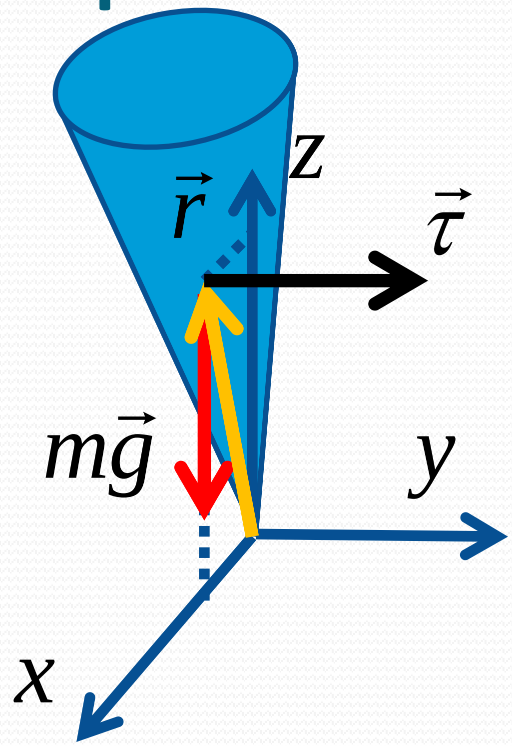
- The total angular momentum of a system is constant if the net external torque acting on the system is zero
  - Angular momentum is a vector, so this means that both direction and the magnitude are conserved
- Examples:
  - Spinning skaters
  - Falling cats
  - Collapsing stars



$$\vec{\tau} = \vec{r} \times m\vec{g}$$

The Motion of Tops





the resulting motion is slow rotation about *z* axis (precession)

tilt in *zy* plane (about *x* axis) causes torque along *x* axis which tries to tilt the top about *y* axis

tilt in *zx* plane (about *y* axis) causes torque along *y* axis which brings the top back to vertical