Chapter 13

Universal Gravitation

Newton's Law of Universal Gravitation

- Every particle in the Universe attracts every other particle

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$G = 6.674\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$

  - Gravitational force acts at a distance without any media
  - There is no repulsive gravitational force

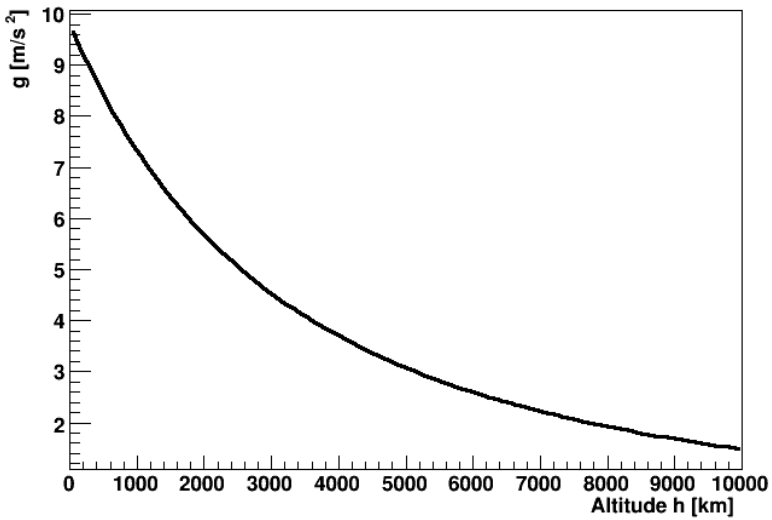
Free-Fall Acceleration and the Gravitational Force

$$F_g = G\frac{mM_E}{r^2} = mg$$

$g_0 = 9.80\ \text{m/s}^2$
$R_E = 6.37\times10^6\ \text{m}$

$$g = G\frac{M_E}{r^2} = g_0\frac{R_E^2}{(R_E + h)^2}$$

Using G, R_E, and g, one can figure out the Earth mass (and density)



Kepler's Laws

- Planets move in ellipses with the Sun in the focus
- A line joining the planet and the Sun sweeps out equal areas in equal time intervals
- The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of the ellipse

Gravitational Field

- A "source" particle creates a "field" – something defined everywhere in space

$$\vec{g} = G\frac{M}{r^2}\vec{r}$$

unit vector (unitless)

- Each particle in the field ("test" particle) experiences a force due to the field created by the source particle

$$\vec{F}_g = m\vec{g}$$

Gravitational Potential Energy

- Gravitational force is conservative → there is associated potential energy

$$U = -G\frac{Mm}{r}$$

obtained by integration of gravitational force

- If change in distance is negligible compared to the distance itself ($\Delta r << r$), then

$$\Delta U = G\frac{Mm}{r^2}\Delta r = mg\Delta r$$

~constant

Total Energy in Planetary Motion

- It is negative for a bound system

$$E = K + U = \frac{1}{2}mv^2 - G\frac{Mm}{r} = \text{const}$$

- For a circular orbit,

$$m\frac{v^2}{r} = G\frac{Mm}{r^2} \qquad \frac{1}{2}mv^2 = G\frac{Mm}{2r} \qquad E = -G\frac{Mm}{2r}$$

$$v = \sqrt{\frac{GM}{r}}$$

also valid for elliptical orbits,
r=semimajor axis length

Escape Speed

- Escape speed = minimum speed for an object to approach the infinite separation
  - At $r=\infty$, $v=0$, so v_{esc} can be found from $E=0$

$$0 = \frac{1}{2}mv_{esc}^2 - G\frac{Mm}{r}$$

planet's mass

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

planet's radius

$$v_{esc} = v\sqrt{2}$$

Black Holes

- In a normal star, gravitational force trying to squeeze it is compensated by pressure produced by nuclear reactions
- When the star runs out of fuel, gravitation prevails and the star collapses
  - the smaller its size, the stronger the gravitation
- If the star is small (like the Sun), the gravitation gets compensated by repulsive forces among electrons
  - the star becomes a white dwarf
- If the star is medium, electrons are squeezed into protons and form neutrons. The gravitation is compensated by quantum degeneracy pressure due to Pauli exclusion principle
  - the star becomes a neutron star
- If the star is large, there is nothing to withstand the gravitation
  - the star collapses into a point and becomes a black hole
- In addition to BHs created from stars, there can be primordial BHs and even home made BHs (LHC?)

Event Horizon

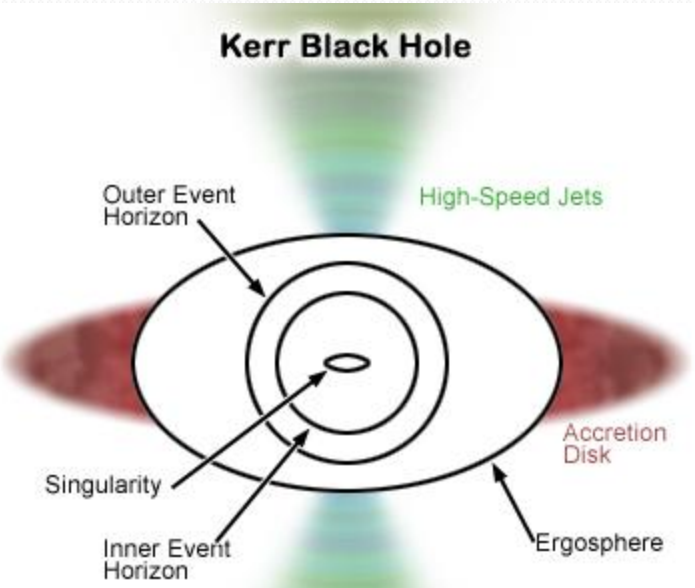
- Since the largest velocity is the speed of light c, nothing can escape the black hole below certain distance (event horizon)
  - this is the point of no return

$$c = \sqrt{\frac{2GM}{r}} \qquad r = \frac{2GM}{c^2} \approx 3\frac{M}{M_{\text{Sun}}}\ \text{km}$$

true for BH with zero charge and angular momentum

Properties of Black Holes

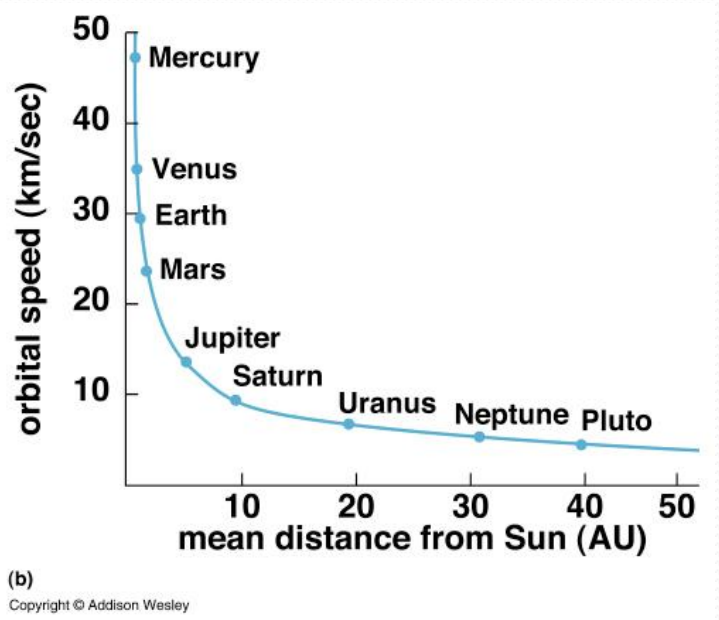
- The only physical part of a BH is “singularity” -- the central part of the BH with zero size and infinite density
- Spinning BHs have “ergosphere” – the region where everything is rotating (to stay still, one would need to move the other way with speed $v>c$)
- BHs can be observed in space as they disturb the stars and clouds around them



Dark Matter

- In the solar system, the farther the planet is, the slower it moves

$$v = \sqrt{\frac{GM}{r}}$$



- Not true for objects in galaxies!
  - v is larger than predicted
    - M is more than accounted for
  - v is flat vs r
    - M grows with r, so it fills the whole space

What is Dark Matter?

- Nobody knows
- A possible candidate: WIMP (weakly interacting massive particles)
  - LHC may be able to see them
  - AMS announced on 3/30/2013 an excess of positrons in cosmic rays – these may be coming from WIMPs