Chapter 2

Motion in One Dimension

Position, Displacement, and Distance

- Position x = location w.r.t. to a chosen reference point
- Displacement = difference between final and initial positions of a particle

$$\Delta x = x_f - x_i$$

- Distance = length of a path followed by a particle
  - Not the same as the distance between two points
  - Not the same as the magnitude of displacement

distance

position

Speed and Velocity

- Speed: scalar (= distance / time)
  - scalars have magnitude
- Velocity: vector (= displacement / time)
  - vectors have magnitude and direction
- Both have the same units [L]/[T]

Average Velocity and Speed

- Average velocity and speed are always defined w.r.t. to a certain time interval Δt

$$\vec{v}_{avg} = \frac{\vec{d}}{\Delta t} \qquad v_{x\,avg} = \frac{\Delta x}{\Delta t} \qquad v_{avg} = \frac{d}{\Delta t}$$

- Average velocity / speed are poor characteristics of particle's motion
  - e.g. if average velocity is zero, the particle might still move during Δt

Instantaneous Velocity and Speed

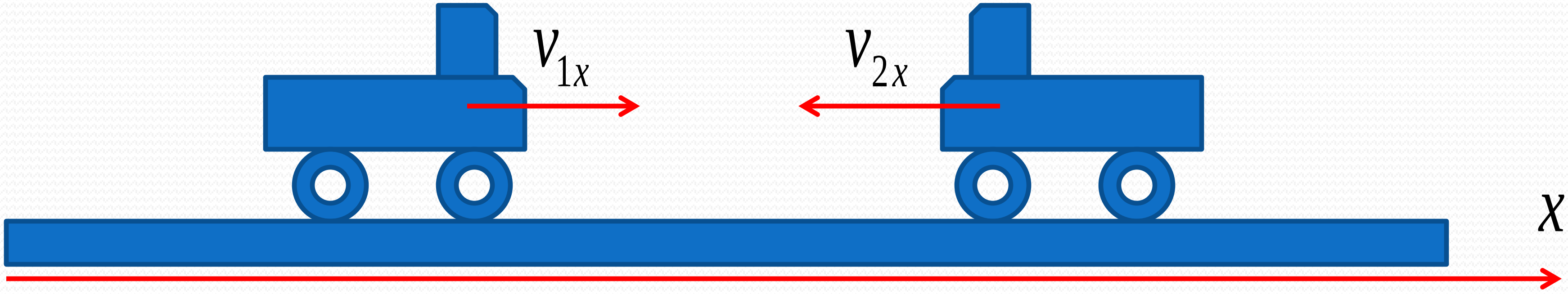
- We want to specify particle's velocity and speed at any given moment in time, just like we can do it for particle's position
- We can do it by considering smaller and smaller time intervals Δt and eventually taking it to the limit Δt→o

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{d}}{\Delta t} \qquad v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

it's notation for derivative, not d times x !

- Unlike average speed, instantaneous speed is equal to the magnitude of instantaneous velocity speed

$v_{1x}=+40$ mi/h
$v_{2x}=-40$ mi/h
$v_1=v_2=40$ mi/h



Particle Under Constant Velocity

- If instantaneous velocity v_x is constant, the average velocity over any time interval is equal to v_x
- Assuming $t_f=t$, $t_i=0$, $\Delta t=t_f-t_i=t$,

$$x_f = x_i + v_x t$$

Acceleration

- If particle's velocity changes with time, the particle is called to be accelerating

$$\Delta v_x = v_{xf} - v_{xi}$$

- We can introduce average acceleration and instantaneous acceleration in the same way as for velocity

$$a_{x\,avg} = \frac{\Delta v_x}{\Delta t} \qquad a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

Particle Under Constant Acceleration

- If instantaneous acceleration a_x is constant, the average acceleration over any time interval is equal to a_x
- Assuming $t_f=t$, $t_i=0$, $\Delta t=t_f-t_i=t$,

$$v_{x\,f} = v_{xi} + a_x t$$

- For velocity which linearly depends on time,

$$v_{x\,avg} = \frac{v_{x\,f} + v_{xi}}{2} \qquad x_f = x_i + v_{x\,avg}t = x_i + v_{x\,i}t + \frac{1}{2}a_x t^2$$

$$t = \frac{v_{x\,f} - v_{xi}}{a_x} \qquad v_{x\,f}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Freely Falling Objects

- Freely falling = moving under influence of gravity alone
  - no engines / springs / etc
  - no air resistance
  - it's OK to have initial velocity ≠0
- All freely falling objects have the same motion: constant acceleration

$$g = 9.80 \text{ m/s}^2$$

Derivation of Kinematic Equations

$$a_x = \frac{dv_x}{dt}$$

$$dv_x = a_x dt$$

$$v_x = \int a_x dt = v_{xi} + a_x t$$

$$v_x = \frac{dx}{dt}$$

$$dx = v_x dt = (v_{xi} + a_x t)dt$$

$$x = \int v_x dt = x_i + v_{xi}t + a_x \frac{t^2}{2}$$