Chapter 3

Vectors
Vectors and Scalars

- Scalar have magnitude
  - example: distance
- Vectors have magnitude and direction
  - example: displacement
Vector Components

- Vector = “arrow”
  - characterized by magnitude and direction
- Vector = set of components
  - characterized by 1, 2, or 3 numbers
- Vector components depend on coordinate system
  - a vector can have different components in different coordinate systems, it doesn’t affect the vector itself
Vector Operations

- They constitute what is called “vector algebra”
- Basic operations:
  - add two vectors
  - multiply a vector by a number
  - negative of a vector: \(-A = (-1) \times A\)
  - subtract vectors: \(A - B = A + (-B)\)
- Basic properties: \(A + B = B + A\), \(A + (B + C) = (A + B) + C\), \(a(bA) = (ab)A\)
- Zero vector: vector of zero magnitude (and undefined direction)
  - \(0 = A - A\) (\(A\) = any vector)
  - \(0 = 0 \times A\) (\(A\) = any vector)
Vector Space

- Vectors of the same kind form “vector space”
  - Vector operations are defined for all vectors in the space
  - Results of vector operations are vectors from the same space
- In general, not all vectors belong to the same space, for example:
  - displacement and force belong to different spaces
  - 1D and 2D vectors belong to different spaces
Linear Combinations of Vectors

- For any vectors $A_1, \ldots, A_m$ and any numbers $a_1, \ldots, a_m$
  \[ a_1A_1 + \ldots + a_mA_m \]
  is also a vector
  - it’s called “linear combination” of vectors $A_1, \ldots, A_m$

- In an $N$-dim vector space, one can pick $N$ vectors of length 1 ("unit vectors") such that any vector from the space can be expressed as a linear combination of these vectors
  - The choice of unit vectors is not unique
  - As soon as the unit vectors are fixed, the coefficients in the linear combination are unique