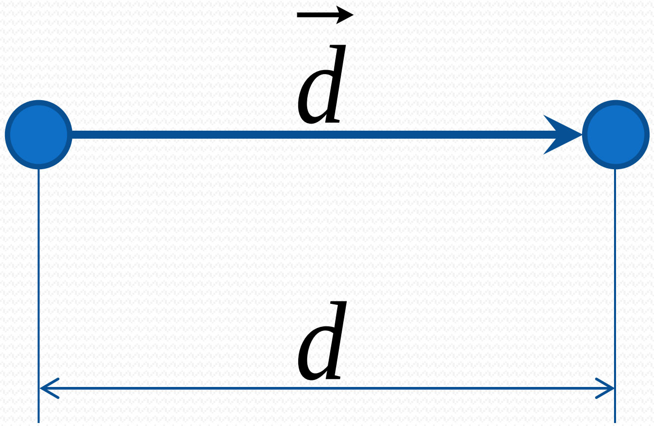
Chapter 3

Vectors

Vectors and Scalars

- Scalar have magnitude
  - example: distance
- Vectors have magnitude and direction
  - example: displacement



Vector Components

- Vector = “arrow”
  - characterized by magnitude and direction
- Vector = set of components
  - characterized by 1, 2, or 3 numbers
- Vector components depend on coordinate system
  - a vector can have different components in different coordinate systems, it doesn’t affect the vector itself

Vector Operations

- They constitute what is called "vector algebra"
- Basic operations:
  - add two vectors
  - multiply a vector by a number
  - negative of a vector: $-A = (-1) \times A$
  - subtract vectors: $A - B = A + (-B)$
- Basic properties: $A+B = B+A$, $A+(B+C)=(A+B)+C$, $a(bA)=(ab)A$
- Zero vector: vector of zero magnitude (and undefined direction)
  - $0 = A-A$ (A = any vector)
  - $0 = 0 \times A$ (A = any vector)

Vector Space

- Vectors of the same kind form “vector space”
  - Vector operations are defined for all vectors in the space
  - Results of vector operations are vectors from the same space
- In general, not all vectors belong to the same space, for example:
  - displacement and force belong to different spaces
  - 1D and 2D vectors belong to different spaces

Linear Combinations of Vectors

- For any vectors $A_1,\ldots,A_m$ and any numbers $a_1,\ldots,a_m$ $a_1A_1+\ldots+a_mA_m$ is also a vector
  - it's called "linear combination" of vectors $A_1,\ldots A_m$
- In an N-dim vector space, one can pick N vectors of length 1 ("unit vectors") such that any vector from the space can be expressed as a linear combination of these vectors
  - The choice of unit vectors is not unique
  - As soon as the unit vectors are fixed, the coefficients in the linear combination are unique