Chapter 4

Motion in Two Dimensions

1D → 2D

- Position $x \rightarrow \vec{r} = x\vec{i} + y\vec{j}$
- Average velocity $v_{x\,avg} = \dfrac{\Delta x}{\Delta t} \rightarrow \vec{v}_{avg} = \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{\Delta x}{\Delta t}\vec{i} + \dfrac{\Delta y}{\Delta t}\vec{j}$
- Velocity $v_x = \dfrac{dx}{dt} \rightarrow \vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\vec{i} + \dfrac{dy}{dt}\vec{j} = v_x\vec{i} + v_y\vec{j}$
- Acceleration $a_x = \dfrac{dv_x}{dt} \rightarrow \vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{dv_x}{dt}\vec{i} + \dfrac{dv_y}{dt}\vec{j} = a_x\vec{i} + a_y\vec{j}$

2D Motion Under Constant Acceleration

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

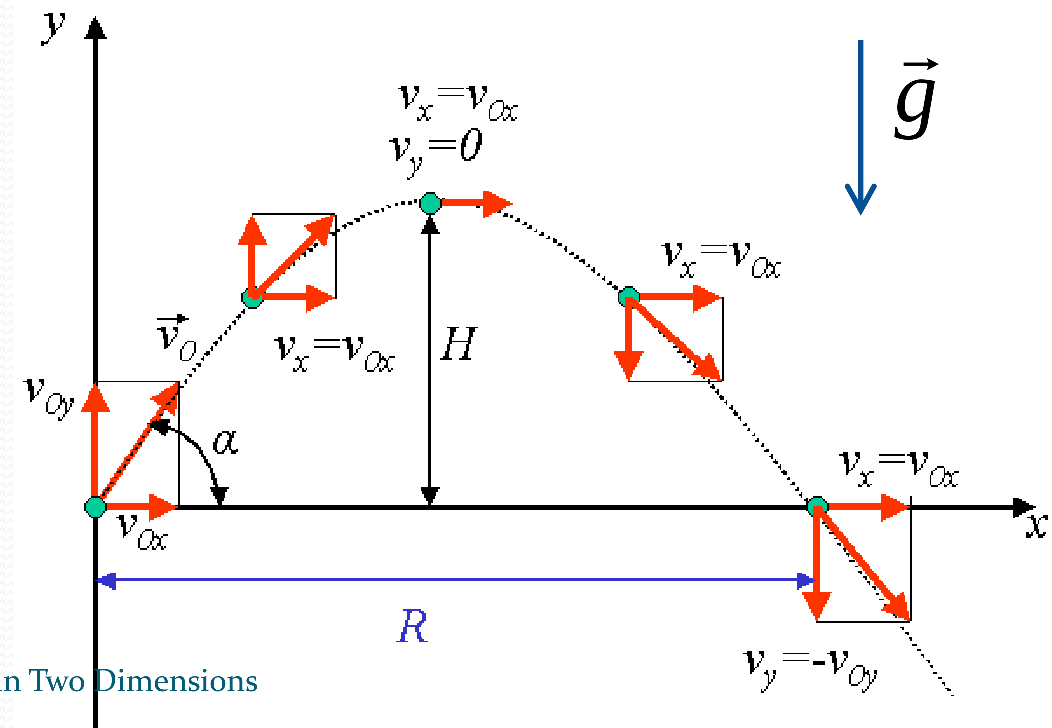
$$v_{xf} = v_{xi} + a_x t$$

$$v_{yf} = v_{yi} + a_y t$$

Projectile Motion

- Assumptions:
  - acceleration is directed downwards and constant
  - air resistance is negligible

R: range
H: maximum height



Projectile Motion

$$x_i = y_i = 0 \qquad v_{xi} = v_i \cos\alpha \qquad v_{yi} = v_i \sin\alpha$$

$$x = v_i \cos\alpha\, t$$

$$y = v_i \sin\alpha\, t - \frac{1}{2} g t^2$$

$$y = x \tan\alpha - \frac{g}{2 v_i^2 \cos^2\alpha} x^2$$

parabola

Projectile Motion

- R, H can be found from $y'(x)=0$

$$R = \frac{v_i^2 \sin 2\alpha}{g} \qquad H = \frac{v_i^2 \sin^2 \alpha}{2g}$$

- Alternatively, R, H can be found from $v_y=0$, where

$$v_y = v_i \sin\alpha - gt$$

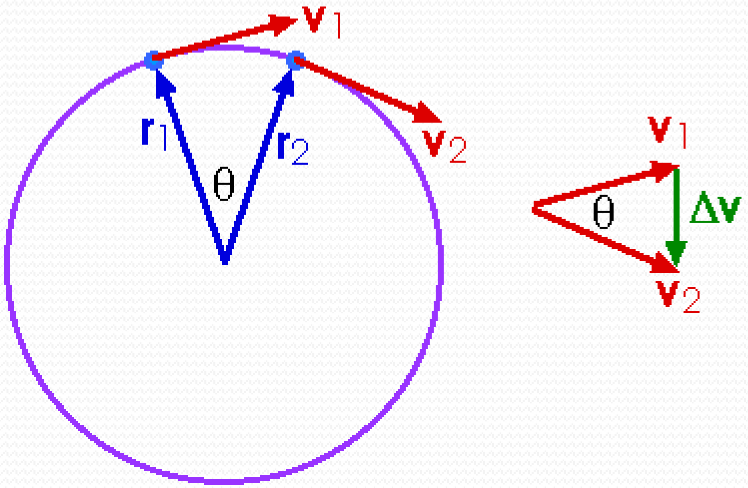
- R is maximal when $\sin 2\alpha=1 \rightarrow \alpha=45°$
- R is the same for α and $90°-\alpha$

Particle in Uniform Circular Motion

- Circular motion = following a circular path
- Uniform circular motion = circular motion with constant speed
- If speed is constant, acceleration is perpendicular to velocity (the component of acceleration parallel to velocity would increase or decrease the speed)
  - acceleration is directed along the radius of the circle
- Since the circular motion repeats itself (i.e. is the same at any moment of time), acceleration magnitude is constant

Particle in Uniform Circular Motion

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$



$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta r}{\Delta t} = \frac{v^2}{r}$$

here $v = |\vec{v}|,\ r = |\vec{r}|,\ \Delta v = |\Delta \vec{v}|,\ \Delta r = |\Delta \vec{r}|$

Relative Velocity and Relative Acceleration

- Problem: relate motion in two reference frames moving w.r.t. each other with a constant velocity
  - Ex.1: a man walking on a beltway; observer A is on the ground, observer B is on the beltway
  - Ex.2: a boat crossing a river; observer A is on the bank, observer B is in the water
- Solution: Galilean transformation equations

$$\vec{r}_A = \vec{r}_B + \vec{v}_{BA} t$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{BA}$$

assume that origins of the two reference frames coincide in space at t=0