Chapter 9

Linear Momentum and Collisions
Linear Momentum

- Definition: \( \vec{p} = m \vec{v} \)
  - momentum is a vector

\[
\vec{F}_{net} = m \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}
\]

- Momentum of an isolated system of particles is conserved

\[
0 = \vec{F}_{12} + \vec{F}_{21} = m_1 \vec{a}_1 + m_2 \vec{a}_2 = \frac{d}{dt} (m_1 \vec{v}_1 + m_2 \vec{v}_2)
\]

\[
\vec{p}_1 + \vec{p}_2 = \text{const}
\]
Momentum vs Energy

- Energy is a scalar, momentum is a vector
- For an isolated system, energy is conserved provided all forces are conservative; momentum is conserved no matter what kind of forces are there
Momentum of Nonisolated System

- If there is an external net force acting on a particle, its momentum will change
  \[ d\vec{p} = \vec{F} \, dt \]
  \[ \Delta \vec{p} = \int \vec{F} \, dt = \vec{I} \]
  - \( \vec{I} \) = impulse of the net force \( \vec{F} \)

- Impulse approximation: situation when the force acts for a short time but is much larger than any other force in the system (typical for collisions)
  - such a force is called impulsive

- It’s often convenient to talk about average force
  \[ \vec{F}_{avg} = \frac{1}{\Delta t} \int \vec{F} \, dt \]
Elastic Collisions

- Elastic collision: both total kinetic energy and momentum of the system is the same before and after collision

\[
m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}
\]

\[
\frac{m_1 v^2_{1i}}{2} + \frac{m_2 v^2_{2i}}{2} = \frac{m_1 v^2_{1f}}{2} + \frac{m_2 v^2_{2f}}{2}
\]
Inelastic Collisions

- If kinetic energy is lost in the collision, it’s called inelastic
  - in general, not enough information to calculate motion after collision
- If colliding objects stick together the collision is called perfectly inelastic

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \]
The Center of Mass

two particles

\[ x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]

many particles

\[ x_{CM} = \frac{1}{M} \sum_i m_i x_i \]

extended object

\[ x_{CM} = \frac{1}{M} \int x \, dm \]

\[ \vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \]

\[ \vec{r}_{CM} = \frac{1}{M} \sum_i m_i \vec{r}_i \]

\[ \vec{r}_{CM} = \frac{1}{M} \int \vec{r} \, dm \]

\[ M = \sum_i m_i \]

\[ M = \int dm \]
Motion of the Center of Mass

- Total linear momentum of the system:

\[ M\vec{v}_{CM} = \sum_{i} m_i \vec{v}_i = \sum_{i} \vec{p}_i = \vec{p}_{tot} \]

- Acceleration of c.m.:

\[ Ma_{CM} = \sum_{i} m_i \vec{a}_i = \sum_{i} \vec{F}_i \]

net force on particle \( i \)
Rocket Propulsion

- This doesn’t work – it’s wrong: \[ F = \frac{d(mv)}{dt} = v \frac{dm}{dt} + m \frac{dv}{dt} \]

- This one is correct:

\[ (m + dm)v = m(v + dv) + dm(v - v_e) \]

\[ m \, dv = v_e \, dm \]

\[ v = v_e \int \frac{dm}{m} \quad v_f - v_i = v_e \ln \frac{m_i}{m_f} \]

exhaust speed