Chapter 9

Linear Momentum and Collisions

Linear Momentum

- Definition: $\vec{p} = m\vec{v}$
  - momentum is a vector

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

- Momentum of an isolated system of particles is conserved

$$0 = \vec{F}_{12} + \vec{F}_{21} = m_1\vec{a}_1 + m_2\vec{a}_2 = \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2)$$

$$\vec{p}_1 + \vec{p}_2 = \text{const}$$

Momentum vs Energy

- Energy is a scalar, momentum is a vector
- For an isolated system, energy is conserved provided all forces are conservative; momentum is conserved no matter what kind of forces are there

Momentum of Nonisolated System

- If there is an external net force acting on a particle, its momentum will change

$$d\vec{p} = \vec{F}dt \qquad \Delta\vec{p} = \int \vec{F}dt = \vec{I}$$

  - I=impulse of the net force F
- Impulse approximation: situation when the force acts for a short time but is much larger than any other force in the system (typical for collisions)
  - such a force is called impulsive
- It's often convenient to talk about average force

$$\vec{F}_{avg} = \frac{1}{\Delta t}\int \vec{F}dt$$

Elastic Collisions

- Elastic collision: both total kinetic energy and momentum of the system is the same before and after collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2}$$

Inelastic Collisions

- If kinetic energy is lost in the collision, it's called inelastic
  - in general, not enough information to calculate motion after collision
- If colliding objects stick together the collision is called perfectly inelastic

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

The Center of Mass

two particles | many particles | extended object

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{CM} = \frac{1}{M} \sum_i m_i x_i$$

$$x_{CM} = \frac{1}{M} \int x \, dm$$

$$\vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{CM} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{r}_{CM} = \frac{1}{M} \int \vec{r} \, dm$$

$$M = \sum_i m_i$$

$$M = \int dm$$

Motion of the Center of Mass

- Total linear momentum of the system:

$$M\vec{v}_{CM} = \sum_i m_i \vec{v}_i = \sum_i \vec{p}_i = \vec{p}_{tot}$$

- Acceleration of c.m.:

$$M\vec{a}_{CM} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i$$

net force on particle *i*

Rocket Propulsion

- This doesn't work – it's wrong: $F = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$
- This one is correct:

$$(m + dm)v = m(v + dv) + dm(v - v_e)$$

$$m\,dv = v_e\,dm$$

$$v = v_e \int \frac{dm}{m} \qquad v_f - v_i = v_e \ln \frac{m_i}{m_f}$$

exhaust speed