Physics 2314, Homework #8 (due 4/4)

Multiple choice questions

M1 If vectors \( \mathbf{A} \) and \( \mathbf{B} \) are perpendicular then vector \( (\mathbf{A} \times \mathbf{B}) \times \mathbf{A} \) is

A. parallel to vector \( \mathbf{A} \).
B. parallel to vector \( \mathbf{B} \).
C. perpendicular to both vectors \( \mathbf{A} \) and \( \mathbf{B} \).
D. zero.

M2 A solid sphere and a hollow sphere have the same mass and radius. They are rotating with the same angular speed about axes passing through their centers. Which one has the higher angular momentum?

A. The solid sphere.
B. The hollow sphere.
C. Their angular momenta are the same.
D. It is impossible to determine.

M3 A system consists of a block of mass \( m \) attached to a cord that is wound around a pulley. The pulley is a solid disk of the same mass \( m \) as the block. The system is originally held at rest so that the block is at height \( H \) above the ground, and then released. When the block hits the ground, its final velocity is \( v_f \).

(a) If \( H \) is increased by a factor of 4, the final velocity \( v_f \)

A. increases by a factor of 4.
B. increases by a factor of 2.
C. increases by a factor of \( \sqrt{2} \).
D. remains the same.
E. decreases by a factor of \( \sqrt{2} \).
F. decreases by a factor of 2.
G. decreases by a factor of 4.

(b) If the mass of the block is decreased by a factor of 4, the final velocity \( v_f \)

A. increases by a factor of 4.
B. increases by a factor of 2.
C. increases by a factor of \( \sqrt{2} \).
D. remains the same.
E. decreases by a factor of \( \sqrt{2} \).
F. decreases by a factor of 2.
G. decreases by a factor of 4.
Problems

P1 Consider vectors $\mathbf{A} = \mathbf{i} + \mathbf{j}$ and $\mathbf{B} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

(a) Find $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$.

(b) Show that $(\mathbf{A} \cdot \mathbf{B})^2 + |\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2 |\mathbf{B}|^2$

(c) Is (b) true for any vectors $\mathbf{A}$ and $\mathbf{B}$? Why?

P2 A conical pendulum consists of a ball of mass 0.05 kg suspended from a string of length 1.1 m. If the angle between the string and the vertical is $30^\circ$, find the angular momentum of the ball about the vertical dashed line.

P3 A particle with mass 1 kg is moving in the $xy$ plane. Its velocity as a function of time is given by

$$\mathbf{v} = 6t^2 \mathbf{i} + 2t \mathbf{j}$$

where $\mathbf{v}$ is in meters per second and $t$ is in seconds. Find the angular momentum of the particle at $t = 2$ s.

P4 A disk of radius 0.1 m and mass 0.2 kg rotates about a frictionless vertical axle with angular speed 1 rad/s. A second disk of radius 0.05 m and mass 0.05 kg, initially not rotating, drops onto the first disk. Because of the friction between the surfaces, the two eventually reach the same angular speed $\omega_f$. Using the principle of conservation of angular momentum, calculate $\omega_f$. 