## HW1, PHYS 3113

P1 (1.16)

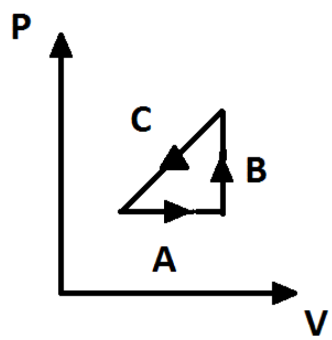
(1) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass $m$ of the air molecules. Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P,$$

called the barometric equation.

(2) Assuming that the temperature of the atmosphere is independent of height, solve the barometric equation to obtain the pressure as a function of height: $P(z) = P(0)\exp(-mgz/kT)$. Show also that the density obeys a similar equation.

P2 (1.33) An ideal gas is made to undergo the cyclic process shown in figure below. For each of the steps A, B, and C, determine whether each of the following is positive, negative, or zero: (a) the work one on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does the process accomplish?




P3 (1.40) In Problem P1, you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however,the temperature of the bottommost 10–15 km of the atmosphere (called the troposphere) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $dT/dz$ exceeds a certain critical value, convection will occur. Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must remain warmer than the surrounding air despite the adiabatic cooling.

(a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}$$

(b) Using the result of (a), find a formula for $dT/dz$. Assume that $dT/dz$ is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. The result should be a constant, independent of temperature and pressure, which evaluates to approximately $-10°C/km$. This fundamental meteorological quantity is known as the dry adiabatic lapse rate.