Physics 3513, Final exam

Each problem consists of two parts, first (required) and second (optional, marked with an asterisk). To get the full grade, you are only required to complete the first part. The second part counts as a bonus (up to +25%).

(1) Solve $z^2 = \bar{z}$.

(∗) Solve $z + iz = |4z^4|$.

(2) Find the exponential Fourier transform $g(\omega) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty} f(x)e^{-i\omega x}\,dx$ of the function

$$f(x) = \begin{cases} \dfrac{1}{2a}, & -a < x < a, \\ 0, & \text{otherwise.} \end{cases}$$

Be sure to express the result in terms of the real functions.

(∗) Find $\displaystyle\lim_{a\to 0}\int_{-\infty}^{+\infty} f(x)e^{-x^2}\,dx$.

(3) Let $A_1 = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$, $A_2 = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$. Show that $\mathbf{e} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is an eigenvector of both A_1 and A_2. What are the corresponding eigenvalues?

(∗) Show that $\mathbf{e}$ is an eigenvector of $A = (A_1 + A_2)^{10}$. What is the corresponding eigenvalue?

(4) Solve $y'' - y = e^{2x} - 1$.

(∗) Solve $(y'' - y)^2 + 2(y'' - y) = e^{2x} - 1$.

(5) Find the interval of convergence of $S(x) = \displaystyle\sum_{n=1}^{\infty} \frac{2^n + 1}{2^n n} x^n$. Be sure to investigate the endpoints of the interval.

(∗) Find $S(-1)$.

(6) Gas law for one mole of ideal gas at temperature T, pressure p, and volume V is $pV = RT$, where R is the gas constant. Show that $\left(\dfrac{\partial p}{\partial V}\right)_T \left(\dfrac{\partial V}{\partial T}\right)_p \left(\dfrac{\partial T}{\partial p}\right)_V = -1$.

(∗) If $F(x, y, z) = 0$, show that $\left(\dfrac{\partial x}{\partial y}\right)_z \left(\dfrac{\partial y}{\partial z}\right)_x \left(\dfrac{\partial z}{\partial x}\right)_y = -1$.