

Physics 3513, Homework #10 (due 12/3)

The numbers in parentheses after the problem number indicate points for each problem.

P1(20) (Boas 5.7, 6.14)

(a) Expand in a Fourier series: $f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 < x < \pi. \end{cases}$

(b) Using the previous result, find $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

P2(40)

(a) Expand in a Fourier series: $f(x) = |x|$, $-\pi < x < \pi$.

(b) Expand in a Fourier series: $f(x) = x^2$, $-\pi < x < \pi$.

(c) Using the previous results, show that $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2} = \frac{3x^2 - 6\pi x + 2\pi^2}{12}$, $0 \leq x \leq \pi$.

(d) Show that (c) can also be obtained by integrating the equality $\sum_{n=1}^{\infty} \frac{\sin nx}{n} = \frac{\pi - x}{2}$.

P3(10) Write $f(x) = e^x$ as the sum of an even function and an odd function.

P4(10) (Boas 9.7) Expand in a Fourier series: $f(x) = \begin{cases} 1, & -1 < x < 1, \\ 0, & -2 < x < -1 \text{ and } 1 < x < 2. \end{cases}$

P5(20) (Boas 12.4, 12.5) Find the exponential Fourier transform of

(a) $f(x) = \begin{cases} 1, & \pi/2 < |x| < \pi, \\ 0, & \text{otherwise;} \end{cases}$

(b) $f(x) = \begin{cases} 1, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$