

Physics 3513, Homework #5 (due 10/1)

The numbers in parentheses after the problem number indicate points for each problem.

P1(10) (Boas 1.5) For $w = 8x^4 + y^4 - 2xy^2$, find $\partial^2 w / \partial x^2$ and $\partial^2 w / \partial y^2$ at the points where $\partial w / \partial x = \partial w / \partial y = 0$.

P2(10) (Boas 2.4) Find the two-variable Maclaurin series for e^{xy} .

P3(10) (Boas 5.4) Given $x = \ln(u^2 - v^2)$, $u = t^2$, $v = \cos t$, find dx/dt .

P4(10) Given $f = f(u, v)$, $u = x - y$, $v = y - z$, find $f_x + f_y + f_z$.

P5(10) (Boas 7.17) If $p^3 + sq = t$, and $q^3 + tp = s$, find $(\partial p / \partial s)_t$, $(\partial p / \partial s)_q$ at $(p, q, s, t) = (-1, 2, 3, 5)$.

P6(10) (Boas 9.11) Find the shortest distance from the origin to the line of intersection of the planes $2x + y - z = 1$ and $x - y + z = 2$.

P7(10) (Boas 12.11) Find $\frac{d}{dx} \int_{3-x}^{x^2} (x-t)dt$ (a) by evaluating the integral first; (b) by differentiating first.

P8(10) Given $f(x, y) = \varphi(x)\psi(y)$ where $\varphi(x)$, $\psi(y)$ are arbitrary functions such that $\varphi(0) = \psi(0) = 1$, find $\frac{f_x f_y}{f_{xy}}$ at the point $x = y = 0$.

P9(10) Find $\frac{dx}{dz}$, $\frac{dy}{dz}$ if $\begin{cases} x + y + z = 0 \\ x^2 + y^2 + z^2 = 1 \end{cases}$.

P10(10) The Schwarzian derivative S of a function $x(t)$ is defined by

$$Sx(t) = \frac{x'''(t)}{x'(t)} - \frac{3}{2} \left(\frac{x''(t)}{x'(t)} \right)^2.$$

Show that the Schwarzian derivative doesn't change under fractional linear transformation $x(t) \rightarrow y(t) = \frac{ax(t) + b}{cx(t) + d}$, that is, $Sy(t) = Sx(t)$.