Physics 3513, Homework #7 (due 10/29)

The numbers in parentheses after the problem number indicate points for each problem.

P1(10) (Boas 3.1) If $\mathbf{A} = (2, -1, -1)$, $\mathbf{B} = (2, -3, 1)$, $\mathbf{C} = (0, 1, 1)$, find $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$, $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$, $(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$.

P2(10) (Boas 3.12) If $\mathbf{A}' = \mathbf{B}\times\mathbf{C}$, $\mathbf{B}' = \mathbf{C}\times\mathbf{A}$, $\mathbf{C}' = \mathbf{A}\times\mathbf{B}$, show that $(\mathbf{A}'\mathbf{B}'\mathbf{C}') = (\mathbf{ABC})^2$.

P3(10) (Boas 6.1) Find the gradient of $w = x^2y^3z$ at $(1, 2, -1)$.

P4(10) (Boas 7.5) Calculate the divergence and the curl of $\mathbf{V} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$.

P5(10) (Boas 8.5) Find the work done by the force $\mathbf{F} = x^2y\mathbf{i} - xy^2\mathbf{j}$ along the straight line joining $(1, 1)$ to $(4, 2)$.

P6(10) (Boas 8.12) Verify that the force $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + \mathbf{k}$ is conservative, and find the corresponding potential.

P7(10) If $f(r)$ is an arbitrary function, $r = \sqrt{x^2 + y^2 + z^2}$, (a) show that $r\,\nabla f(r) = f'(r)\,\mathbf{r}$; (b) verify that the force $\mathbf{F} = f(r)\,\mathbf{r}$ is conservative.

P8(10) (Boas 9.2) Use the Green's theorem to calculate $\oint 2xdy - 3ydx$ around the square with vertices $(0, 2)$, $(2, 0)$, $(-2, 0)$, and $(0, -2)$.

P9(10) (Boas 10.8) Calculate $\iiint \nabla\mathbf{V}\; d\tau$ over the volume $x^2 + y^2 \le 4$, $0 \le z \le 5$, where $\mathbf{V} = \sqrt{x^2 + y^2}(x\mathbf{i} + y\mathbf{j})$.

P10(10) (Boas 11.7) Find $\iint \nabla\times\mathbf{V}\cdot\mathbf{n}d\sigma$ over a surface whose bounding curve is in the (x, y) plane, where $\mathbf{V} = (x - x^2z)\mathbf{i} + (yz^3 - y^2)\mathbf{j} + (x^2y - xz)\mathbf{k}$.