Physics 3513, Midterm I: solutions

(1) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x-1|$, so the series converges if $0 < x < 2$ and diverges if $x < 0$ or $x > 2$. At $x = 0$, both $\sum_{n=1}^{\infty}\frac{(n-1)!}{n!}(-1)^n = \sum_{n=1}^{\infty}\frac{(-1)^n}{n} = -\ln 2$ and $\sum_{n=1}^{\infty}\frac{1}{n!} = e$ converge, so their sum also converges. At $x = 2$, $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges and $\sum_{n=1}^{\infty}\frac{1}{n!}$ converges, so their sum diverges. **Answer:** $0 \le x < 2$.

(2) Evaluating the determinant gives $z^3 = -1$, or $(re^{i\varphi})^3 = e^{i\pi}$, from which we find $r = 1$, $\varphi = \frac{\pi}{3} + \frac{2\pi}{3}k$, $k = 0, 1, 2$. **Answer:** -1, $\frac{1}{2} \pm \frac{\sqrt{3}}{2}$.

(3) Series expantion of the numerator gives $1+2x+4x^2/2+8x^3/6+\cdots-2(1+x+x^2/2+x^3/6+\cdots)+1-x^2+x^4/2-\cdots = x^3+\cdots$. **Answer:** 1.

(4) The characteristic equation $(x-\lambda)^3 - x^2(x-\lambda) = (x-\lambda)\lambda(\lambda-2x) = 0$ has solutions $\lambda = 0, x, 2x$. If $\lambda = 1$ then $x = 1$ or $x = 1/2$. **Answer:** 1, 1/2.

(5) $f_x = 2x + y$, $f_y = x + 2y$, so $f_{xx} + f_{xy} + f_{yy} = 2 + 1 + 2 = 5$. **Answer:** 5.