Physics 3513, Midterm II: solutions

(1) $V = \displaystyle\int_0^1 \pi(x^2 - x^3)^2\, dx = \frac{\pi}{105}$.

(2) $\nabla \times \mathbf{F} = (e^y - e^y)\mathbf{i} + (e^z - e^z)\mathbf{j} + (e^x - e^x)\mathbf{k} = 0$, so the function is conservative. The potential $\phi = -\displaystyle\int_{(0,0,0)}^{(x,y,z)} \mathbf{F}\, d\mathbf{r} = -\left(\int_0^x dx + \int_0^y e^x\, dy + \int_0^z (e^y + xe^z)\, dz\right) = -\left(x + e^x y + e^y z + x\left(e^z - 1\right)\right) = -\left(e^x y + e^y z + e^z x\right)$.

(3) The auxiliary equation $D^2 + 10D + 9 = 0$ has roots -1, -9, so the complimentary function is $y_c = C_1 e^{-x} + C_2 e^{-9x}$. Trying $y_p = \mu e^{-10x}$, we get $9\mu = 9$, so $\mu = 1$. **Answer:** $y = C_1 e^{-x} + C_2 e^{-9x} + e^{-10x}$.

(4) $\nabla(\mathbf{cr}) = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right)(c_x x + c_y y + c_z z) = (c_x, c_y, c_z) = \mathbf{c}$.

(5) $\displaystyle\oint_C -2y\sin^2 x\, dx + \sin x \cos x\, dy = \iint (\cos^2 x - \sin^2 x + 2\sin^2 x)\, dx\, dy = \int_{-\pi/2}^{\pi/2} dx \int_0^1 dy = \pi$.