Final exam, PHYS 5113

P1 A system follows the fundamental equation $F = -a\sqrt{VN}\,T + bV\sqrt{T}$, where a, b are positive constants.

(a) Show that the system is stable.

(b) Derive the fundamental equation in the form $U = U(S, V, N)$.

(c) Calculate the heat capacity at constant volume C_V and the isothermal compressibility κ_T as functions of V, T, N.

(d) Calculate transfer of heat $\delta Q = T\,dS$ at constant temperature T if the volume is changed by a small amount dV,

P2 Consider a system of $\bar{N}$ particles of mass m, each located in an isolated one-dimensional infinite potential well of width L. The energy levels of particles in this system may be written as

$$\varepsilon_n = \frac{n^2 h^2}{8mL^2} = n^2\varepsilon, \quad n = 1, 2, 3, \ldots.$$

(a) In the canonical formalism, write down the partition function Z and the Helmholtz potential F of the system. Assume that the particles are indistinguishable.

(b) Calculate the internal energy U and the heat capacity $C_L = \left(\frac{\partial U}{\partial T}\right)_L$ in the high temperature limit, when the summation can be replaced by an integral. Explain the result in terms of the equipartition theorem.

(c) Calculate U and C_L in the low temperature limit, retaining only the first two terms of the expansion over the small parameter $x = e^{-\beta\varepsilon} \ll 1$

(d) Compare the result of (c) with the one for the two-states model $\varepsilon_n = \begin{cases} 0, & n = 0 \\ \varepsilon, & n = 1 \end{cases}$