Kinematics of particle decays

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Units

- c: speed of light in vacuum, a universal constant

$$c = 299792458 \ \mathrm{m/s}$$

- This is an exact number
  - why? Maybe in the future we will measure it with a better accuracy?

Units

- c is fixed because this is the way the unit of length is defined: the meter is the length of the path traveled by light in vacuum during 1/299792458 of a second
- nobody prevents us from picking a system of units where $c = 1$
  - these units are called "natural" since c is a natural unit for the speed
- in such units, $E = mc^2$ becomes $E = m$: energy and mass are the same
  - more accurately, the rest energy of a particle measured in natural units of energy, is equal to its rest mass measured in natural units of mass
- From Planck relation $E = h\nu$ we conclude that energy and frequency are the same, too! So a natural step is to assume $h = 1$
  - Units where $h = c = 1$ are called Planck units and widely used in HEP

Units in HEP

- The unit of energy used in HEP is called electronvolt (eV)
- 1 eV is the amount of energy gained by the charge of a single electron moved across an electric potential difference of one volt
  - it has simple experimental meaning, and it is very useful when constructing particle accelerators
- eV is not well suited for everyday's use

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ eV}/c^2 = 1.783 \times 10^{-36} \text{ kg}$$

  - it's usual to keep c and c^2 when talking about momenta and energies, i.e. measuring energy in eV/c^2 and momentum in eV/c
  - numerically, it's irrelevant because in Plank units c=1
- But eV works very well in the particle world

$$m_e = 9.1 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}$$

Metric prefixes

- Useful prefixes
  - energy/mass/momentum: 1 MeV$=10^6$ eV, 1 GeV$=10^9$ eV
  - length/time: 1 fm$=10^{-15}$ m, 1 ns$=10^{-9}$ s (equivalent to 0.3 m)
- cross section (measured in square units): 1 pb$=10^{-12}$ b, 1 fb$=10^{-15}$ b, where *b* is the barn: 1 b$=10^{-28}$ m^2

Prefix	Symbol	1000^m	10^n	Decimal	Short scale	Long scale	Since[n 1]
yotta	Y	1000^8	10^{24}	1 000 000 000 000 000 000 000 000	Septillion	Quadrillion	1991
zetta	Z	1000^7	10^{21}	1 000 000 000 000 000 000 000	Sextillion	Trilliard	1991
exa	E	1000^6	10^{18}	1 000 000 000 000 000 000	Quintillion	Trillion	1975
peta	P	1000^5	10^{15}	1 000 000 000 000 000	Quadrillion	Billiard	1975
tera	T	1000^4	10^{12}	1 000 000 000 000	Trillion	Billion	1960
giga	G	1000^3	10^9	1 000 000 000	Billion	Milliard	1960
mega	M	1000^2	10^6	1 000 000	Million		1960
kilo	k	1000^1	10^3	1 000	Thousand		1795
hecto	h	$1000^{2/3}$	10^2	100	Hundred		1795
deca	da	$1000^{1/3}$	10^1	10	Ten		1795
		1000^0	10^0	1	One		–
deci	d	$1000^{-1/3}$	10^{-1}	0.1	Tenth		1795
centi	c	$1000^{-2/3}$	10^{-2}	0.01	Hundredth		1795
milli	m	1000^{-1}	10^{-3}	0.001	Thousandth		1795
micro	μ	1000^{-2}	10^{-6}	0.000 001	Millionth		1960
nano	n	1000^{-3}	10^{-9}	0.000 000 001	Billionth	Milliardth	1960
pico	p	1000^{-4}	10^{-12}	0.000 000 000 001	Trillionth	Billionth	1960
femto	f	1000^{-5}	10^{-15}	0.000 000 000 000 001	Quadrillionth	Billiardth	1964
atto	a	1000^{-6}	10^{-18}	0.000 000 000 000 000 001	Quintillionth	Trillionth	1964
zepto	z	1000^{-7}	10^{-21}	0.000 000 000 000 000 000 001	Sextillionth	Trilliardth	1991
yocto	y	1000^{-8}	10^{-24}	0.000 000 000 000 000 000 000 001	Septillionth	Quadrillionth	1991

1. ^ The metric system was introduced in 1795 with six prefixes. The other dates relate to recognition by a resolution of the CGPM.

Kinematics of particle interactions

- In HEP we are dealing with particles moving with speeds close to c
  - use relativistic mechanics
- To describe kinematics of particle interactions, we need two conservation laws: energy and momentum combined into 4-vector conservation
- Example: collision of two particles (a.k.a. $2 \to 2$ process)

$$\begin{cases} E_a + E_b & = & E_c + E_d \\ \vec{p}_a + \vec{p}_b & = & \vec{p}_c + \vec{p}_d \end{cases}$$

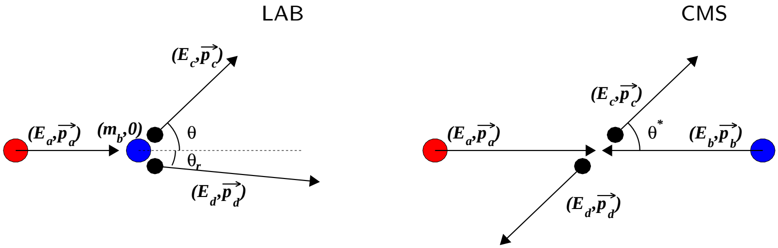
  - compare to classical mechanics: in HEP (total) energy is always conserved (no heat etc), but the particles themselves (and even their number) do not have to be the same before and after the collision
  - a collision is called elastic if the particles before and after the collision are the same
- We assume that particle masses are known

$$E_i = \sqrt{(m_i c^2)^2 + (p_i c)^2},\ p_i = |\vec{p}_i|,\ i = a, b, c, d$$

  - in Planck units, $E_i = \sqrt{m_i^2 + p_i^2}$
- Question: if initial momenta are known, how many parameters remain free?

LAB frame and CMS frame

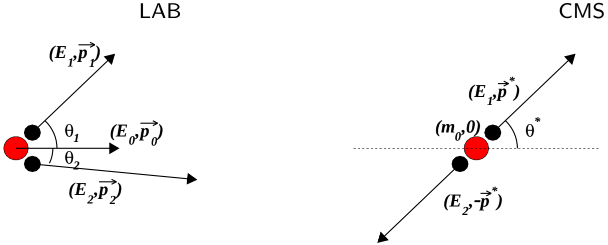
- LAB frame: one particle (target) is at rest
  - *a*: incident particle, *b*: target particle, *c*: scattered particle, *d*: recoil particle
- CMS frame: total momentum of initial particles is zero
  - some confusion here: in collider experiments (LHC) LAB frame for initial colliding particles (protons) is also CMS frame



Particle decays

- Two-body, three-body, *n*-body decays decays
- LAB frame: the one defined by initial collision
- CMS frame: the colliding particle is at rest

$$\begin{cases} E_0 = E_1 + E_2 \\ \vec{p}_0 = \vec{p}_1 + \vec{p}_2 \end{cases}$$



Invariant mass

- Most of the particles are unstable and decay before we can register them
  - how to observe them?
- A straightforward way is to measure 4-vectors of the decay products and calculate their sum
- E.g. for a two-body decay:

$$M = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2} \quad (*)$$

- No matter how particles 1 and 2 are moving, if they result from a decay of a particle of mass M, (*) will always give the same answer for all pairs we probe!

Invariant mass distribution

- In real life, if we apply formula (*) for many particles, the results will be slightly different from measurement to measurement
  - reason 1: measurement errors
  - reason 2: uncertainty principle (particle's finite lifetime)
- One measurement is not enough! Suppose we at CERN want to detect Z bosons via their decays into electron-positron pairs. What would be our plan?
  - sit and watch for e^+e^- pairs coming from proton-proton collisions
  - each time an e^+e^- pair is detected, measure the momenta of e^+ and e^- and calculate their invariant mass m_{ee}
  - calculate the average of all the measurements – this will be the Z mass!

Invariant mass histogram

- A convenient way to present results of many measurements is a "histogram"
  - it is a set of 2-d points where x-coordinate of each point represents certain measured value (m_{ee}) and y-coordinate of the point represents the number of cases when we obtained this value
  - in the previous example, we will have points at $x = 91, y = 1000$; $x = 90, y = 970$; $x = 92, y = 920$; ...
- If properly done, the points will form a bell-shaped curve
  - measurement errors result in a Gaussian curve

$$y = C \exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right)$$

  - uncertainty principle leads to Breit-Wigner shape

$$y = \frac{\Gamma}{(x - E_0)^2 + (\Gamma/2)^2}$$

  - there are other effects which further distort the shape of the distribution

Invariant mass distribution of electron-positron pairs originated from Z boson decays

Origin: ATLAS experiment, Phys.Lett. B705 (2011) 415-434