Passage of particles through matter

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Introduction

- The only way to detect particles is to look at the energy deposited as the particles traverse the detector material
- There are several mechanisms for particles to lose energy
  - they are different for heavy particles and electrons/photons
  - they strongly depend on particle's energy
  - in many cases, there is no strict theory – the energy loss is understood at the phenomenological level
- Items I'm going to cover:
  - energy loss by ionization: Bethe formula
  - energy loss fluctuations
  - δ-rays
  - multiple scattering
  - energy loss by electrons and photons
  - bremsstrahlung
  - pair production
  - Cherenkov and transition radiation
- Reference: Particle Data Group

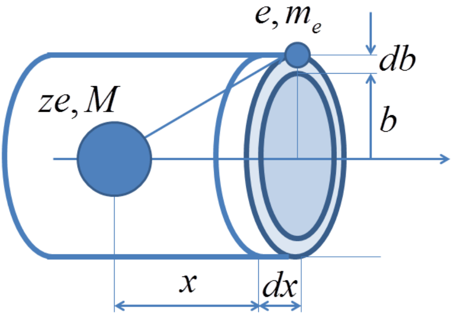
  http://pdg.lbl.gov/2017/reviews/rpp2016-rev-passage-particles-matter.pdf

Energy loss by ionization for heavy charged particles

- this is the most important process for many particle detectors
- we are talking about heavy ($M >> m_e$) charged (ze) particles
- energy loss occurs through elastic collisions with electrons

Classic derivation of Bethe formula

- A heavy charged particle passes an electron at impact parameter b



Classic derivation of Bethe formula

- momentum transfered to the particle:

$$\Delta p_{||} = \int_{-\infty}^{+\infty} F_{||}\, dt = 0 \qquad \Delta p_{\perp} = \int_{-\infty}^{+\infty} F_{\perp}\, dt = \int_{-\infty}^{+\infty} F_{\perp} \frac{dx}{v}$$

- F is the Coulomb's force between the particle and the electron:

$$F = \frac{1}{4\pi\epsilon_0} \frac{ze\, e}{x^2 + b^2} \qquad F_{\perp} = F \sin\theta = F \frac{b}{\sqrt{x^2 + b^2}}$$

- take the integral:

$$\int_{-\infty}^{+\infty} \frac{b\, dx}{(x^2 + b^2)^{3/2}} = \{x = b \tan\xi\} = \frac{1}{b} \int_{-\pi/2}^{+\pi/2} \cos\xi\, d\xi = \frac{2}{b}$$

- the answer:

$$\Delta p_T = \frac{1}{4\pi\epsilon_0} \frac{2ze^2}{bv}$$

Classic derivation of Bethe formula

- energy transfer to a single electron with impact parameter b:
$$\Delta E = \frac{\Delta p^2}{2m_e} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{1}{2m_e}\left(\frac{2ze^2}{bv}\right)^2$$

- energy loss (negative since the energy decreases)
$$-dE = \frac{\Delta p^2}{2m_e}\, n\, 2\pi b\, db\, dx \qquad n \text{ is electron density, } n = \rho Z N_A / A$$

$$-\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{\Delta p^2}{2m_e}\, n\, 2\pi b\, db = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2 e^4}{m_e v^2} n\, 2\pi \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

$$-\frac{dE}{dx} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{4\pi n z^2 e^4}{m_e v^2} \ln \frac{b_{max}}{b_{min}}$$

Classic derivation of Bethe formula

- what are the limits of integration? (can't be zero/∞)
- b_{max}: corresponds to the case when the energy transfer is equal to "typical ionization energy" named mean ionization potential I
  $\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e b_{max}^2 v^2} = I$
- b_{min}: corresponds to a head-on collision between the particle and the electron, in this case the electron acquires speed $2v$:
  $\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e b_{min}^2 v^2} = \frac{1}{2} m_e (2v)^2$, therefore

$$-\frac{dE}{dx} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{4\pi n z^2 e^4}{m_e v^2} 2 \ln \frac{2 m_e v^2}{I}$$

- one can show that factor 2 in front of the log should not be there

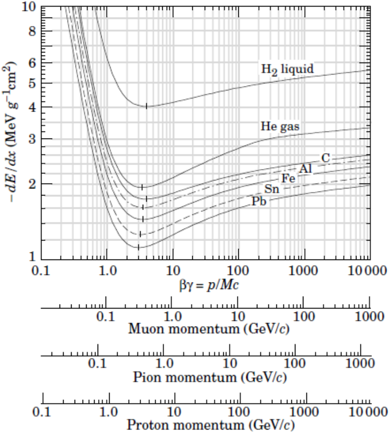
Relativistic Bethe formula

- the derived formula is valid for the non-relativistic case $\beta = v/c << 1$
- if $\beta \sim 1$, then the formula has to be corrected:
$$-\frac{dE}{dx} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{4\pi n z^2 e^4}{m_e c^2 \beta^2} \left(2 \ln \frac{2 m_e c^2 \beta^2}{I(1-\beta^2)} - \beta^2\right)$$
- there is one more fix to the formula – so called density effect (screening of electrons) δ
- the formula quoted in PDG ("mass stopping power"):
$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

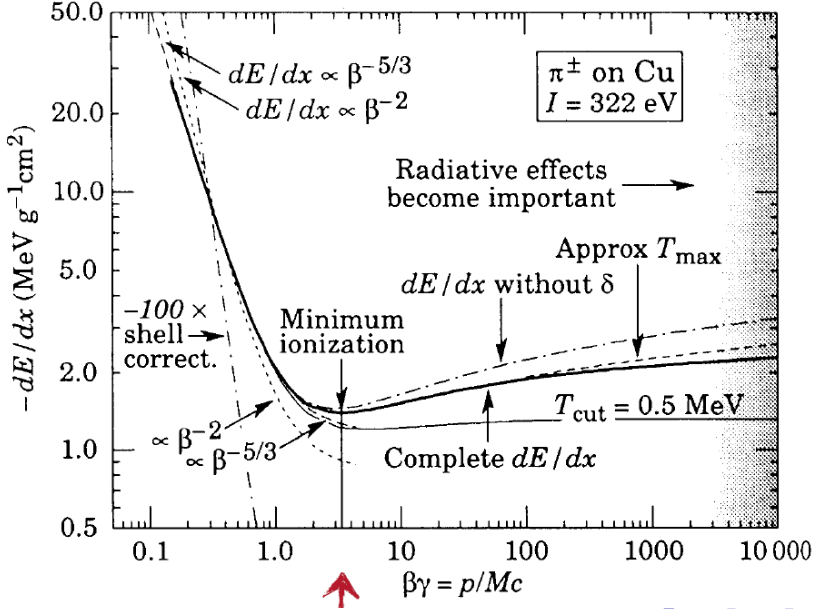
$K = 0.307075 \ \mathrm{MeV\ mol^{-1}\ cm^2}$, A in $\mathrm{g/mol}$
$\gamma = 1/\sqrt{1-\beta^2}$
$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

Features of the Bethe formula

- the formula describes energy losses for $0.1 < \beta\gamma < 1000$ for intermediate Z materials with an accuracy of a few percent
- general features:
  - β^{-2} at low energies
  - minimum energy loss at $\beta\gamma = 3 - 4$
  - saturation at high energies due to density effect
- A and Z dependence



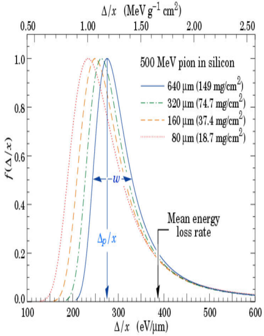
The Bethe formula: pions in Cu



Mean energy loss and fluctuations

"Few concepts in high energy physics are as misused as dE/dx" (PDG)

- the Bethe formula describes mean energy loss
- $\Delta E = \sum_1^N \Delta E_i$, partial energy losses are statistically distributed and fluctuate from measurement to measurement



- in thin absorbers, losses follow the Landau distribution

$$p(x) = \frac{1}{\pi}\int_0^\infty e^{-t\ln t - xt}\sin(\pi t)\,dt$$

- mean and variance of the distribution are infinite (long tail!)