Passage of particles through matter

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Introduction

- The only way to detect particles is to look at the energy deposited as the particles traverse the detector material
- There are several mechanisms for particles to lose energy
  - they are different for heavy particles and electrons/photons
  - they strongly depend on particle's energy
  - in many cases, there is no strict theory – the energy loss is understood at the phenomenological level
- Items I'm going to cover:
  - energy loss by ionization: Bethe formula
  - energy loss fluctuations
  - δ-rays
  - multiple scattering
  - energy loss by electrons and photons
  - bremsstrahlung
  - pair production
  - Cherenkov and transition radiation
- Reference: Particle Data Group

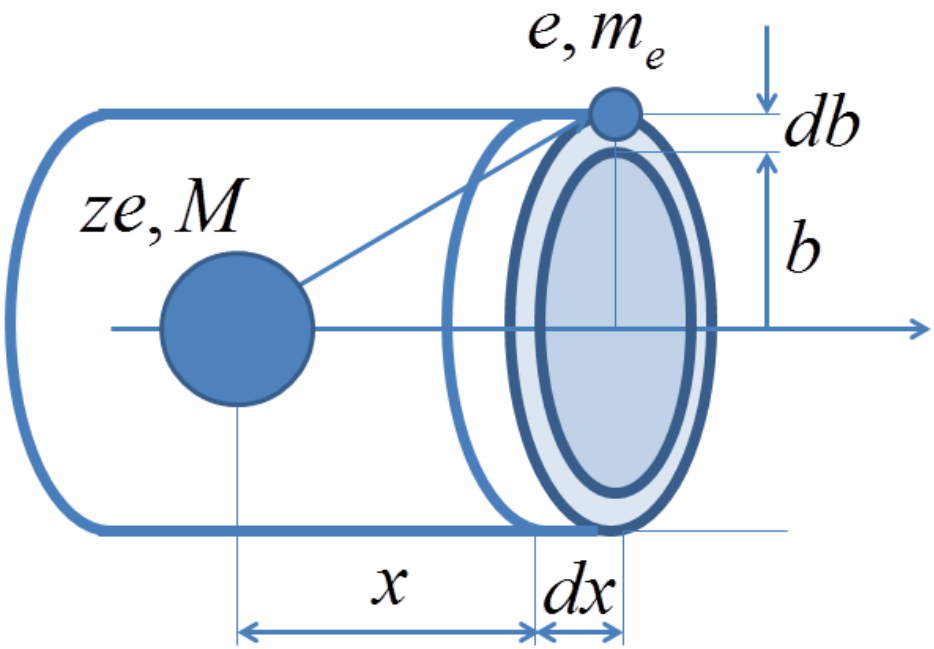
  https://pdg.lbl.gov/2020/reviews/rpp2020-rev-passage-particles-matter.pdf

Energy loss by ionization for heavy charged particles

- Massive ($M \gg m_e$) charged (ze) particles loose their energy mostly due to ionization
  - this is the most important process for many particle detectors
  - energy loss occurs through elastic collisions with electrons
- The mean rate of energy loss for moderately relativistic charged heavy particles can be described by the "Bethe equation"

Classical derivation of Bethe formula

- A heavy charged particle passes an electron at impact parameter b



Classical derivation of Bethe formula

- momentum transfered to the particle:

$$\Delta p_{||} = \int_{-\infty}^{+\infty} F_{||}\, dt = 0 \qquad \Delta p_{\perp} = \int_{-\infty}^{+\infty} F_{\perp}\, dt = \int_{-\infty}^{+\infty} F_{\perp} \frac{dx}{v}$$

- F is the Coulomb's force between the particle and the electron:

$$F = \frac{1}{4\pi\epsilon_0} \frac{ze\, e}{x^2 + b^2} \qquad F_{\perp} = F \sin\theta = F \frac{b}{\sqrt{x^2 + b^2}}$$

- take the integral:

$$\int_{-\infty}^{+\infty} \frac{b\, dx}{(x^2 + b^2)^{3/2}} = \{x = b \tan\xi\} = \frac{1}{b} \int_{-\pi/2}^{+\pi/2} \cos\xi\, d\xi = \frac{2}{b}$$

- the answer:

$$\Delta p_T = \frac{1}{4\pi\epsilon_0} \frac{2ze^2}{bv}$$

Classical derivation of Bethe formula

- energy transfer to a single electron with impact parameter b:

$$\Delta E = \frac{\Delta p^2}{2m_e} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{1}{2m_e}\left(\frac{2ze^2}{bv}\right)^2$$

- energy loss (negative since the energy decreases)

$$-dE = \frac{\Delta p^2}{2m_e}\, n\, 2\pi b\, db\, dx \qquad n \text{ is electron density, } n = \rho Z N_A/A$$

$$-\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{\Delta p^2}{2m_e}\, n\, 2\pi b\, db = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e v^2}\, n\, 2\pi \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

$$-\frac{dE}{dx} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{4\pi n z^2 e^4}{m_e v^2} \ln\frac{b_{max}}{b_{min}} = \rho K z^2 \frac{Z}{A}\frac{1}{\beta^2} \ln\frac{b_{max}}{b_{min}}$$

$K = 0.307075\ \mathrm{MeV\ mol^{-1}\ cm^2}$, A in $\mathrm{g/mol}$

Classical derivation of Bethe formula

- What are the limits of integration? (can't be zero/∞)
  - b_{max}: minimal energy transfer, corresponds to "mean ionization potential" I (averaged excitaion potential of electrons in the target)

$$\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e b_{max}^2 v^2} = I$$

  - b_{min}: maximal energy transfer, corresponds to head-on collision between the particle and the electron

$$\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2z^2e^4}{m_e b_{min}^2 v^2} = T_{\max} = \frac{2m_e c^2\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

  - low energy approximation: $T_{\max} \approx 2m_e c^2\beta^2\gamma^2$

$$-\frac{dE}{dx} = \rho K z^2 \frac{Z}{A}\frac{1}{\beta^2}\frac{1}{2}\ln\frac{T_{\max}}{I}$$

Bethe formula

- Accurate QM derivation:

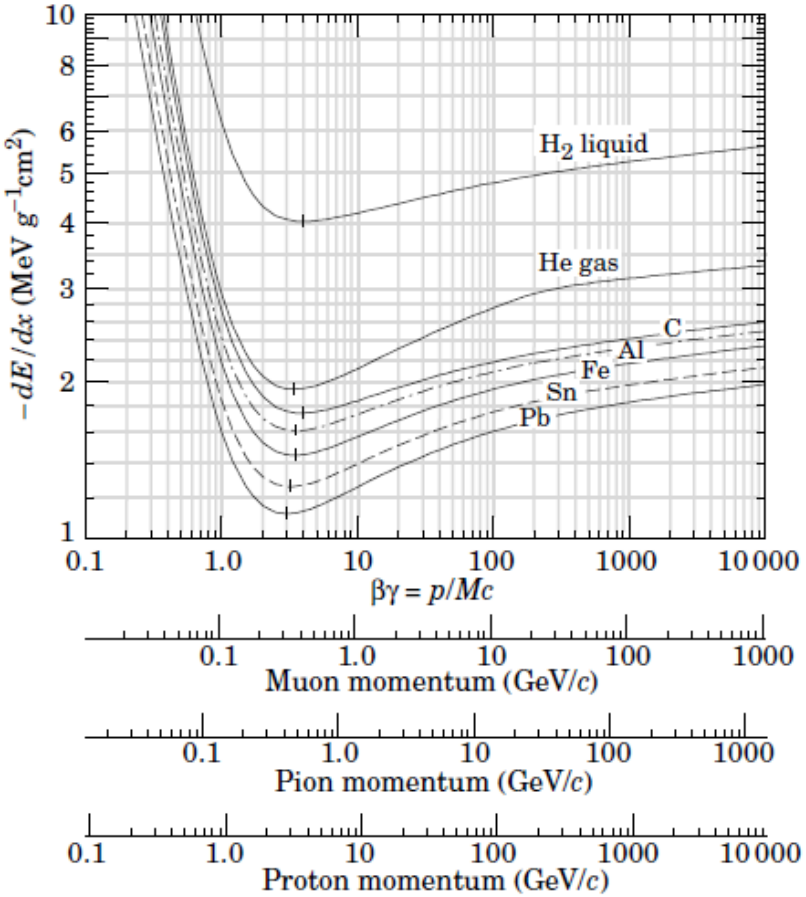
$$-\frac{dE}{dx} = \rho K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

  - note the "density factor" δ, an ad hoc fix to the formula due to screening of electrons

- This is "linear stopping power" (MeV/cm)
  - another commonly used quantity is "mass stopping power" (MeV $g^{-1}cm^2$), obtained by dividing the formula above by density ρ

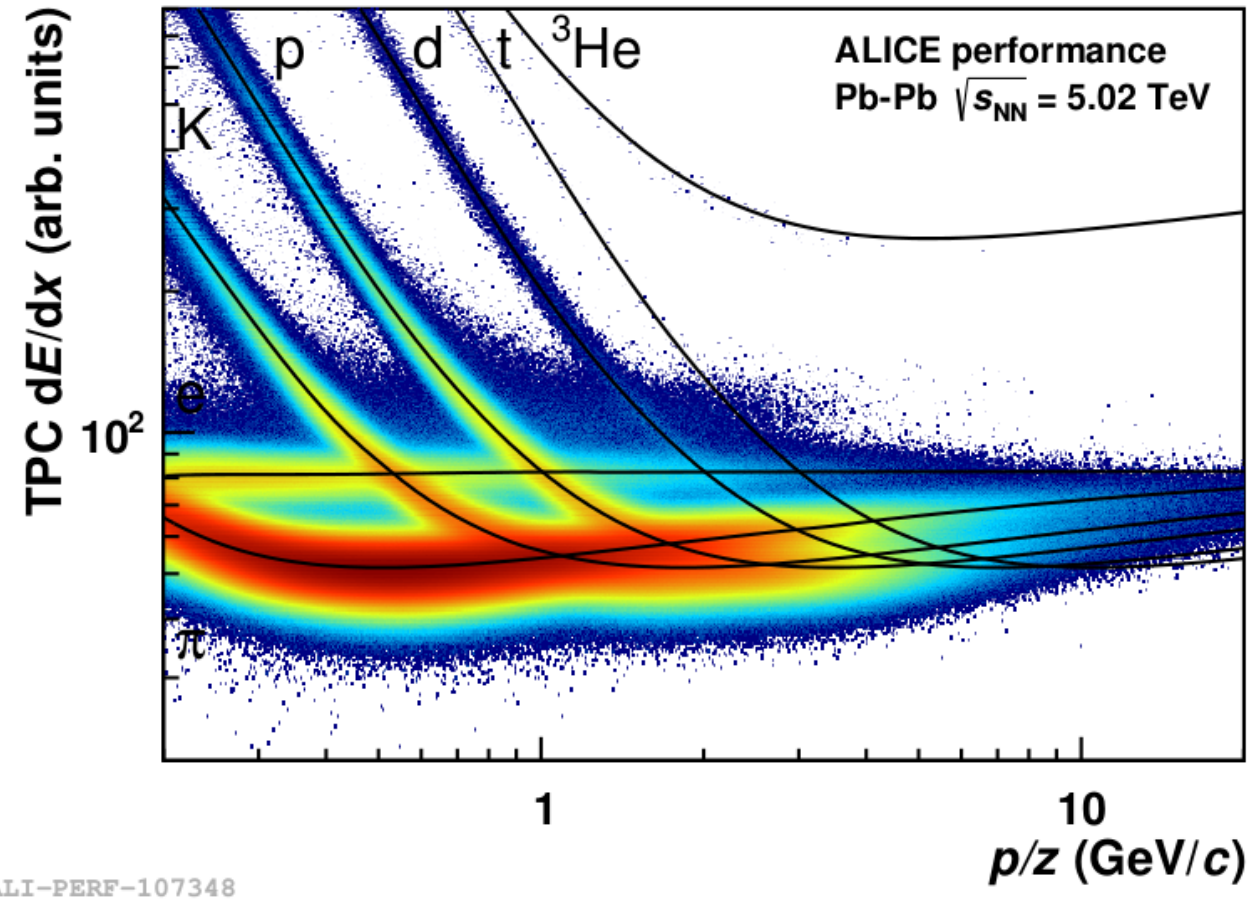
Features of the Bethe formula

- the formula describes energy losses for $0.1 < \beta\gamma < 1000$ for intermediate Z materials with an accuracy of a few percent
- general features:
  - β^{-2} at low energies
  - minimum energy loss at $\beta\gamma = 3 - 4$
  - saturation at high energies due to density effect
- A and Z dependence



The Bethe formula and particle identification

- dE/dx only depends on particle velocity (dependence on mass due to M in $T_{\max}$ is negligible)
  - can be used to discriminate between different types of particles
  - electrons don't follow Bethe formula

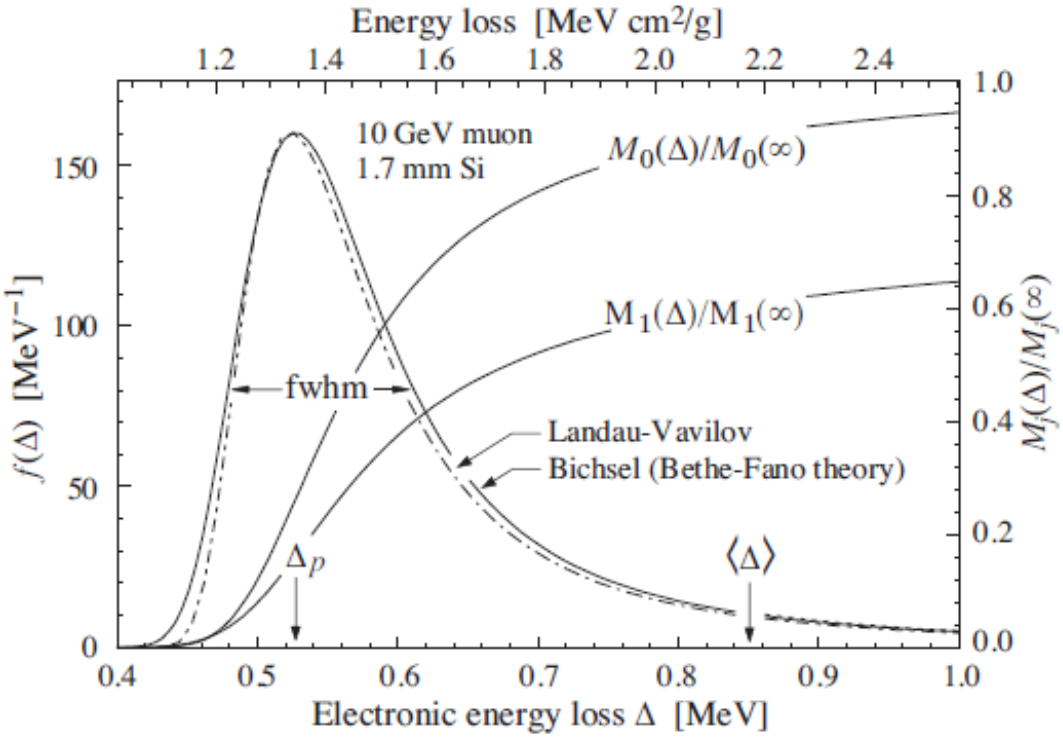


Mean energy loss and fluctuations

- The Bethe formula describes mean energy loss
  - actual energy loss is a statistical process, $\Delta E = \sum_1^N \Delta E_i$, where partial energy losses fluctuate from measurement to measurement
  - is it a good idea to describe energy loss fluctuations with a Gaussian distribution?
- Energy loss in thin detectors can be described by Landau distribution

$$p(x) = \frac{1}{\pi} \int_0^\infty e^{-t \ln t - xt} \sin(\pi t)\, dt$$

- Assumptions of Landau theory:
  - typical energy loss is small compared to the maximum energy loss in a single collision – lifted in Vavilov theory
  - typical energy loss is large compared to the binding energy of the most tightly bound electron – addressed in Urbán model
- Note that $\langle p \rangle = \sigma(p) = \infty$
  - addressed by introducing a maximum to the sampled random variable

Mean energy loss and fluctuations

- Example: a 10 GeV muon traversing 1.7 mm of Si
- Δ_p: most probable energy loss
- $\langle\Delta\rangle$: mean energy loss from Bethe