Passage of particles through matter

Alexander Khanov

PHYS6260: Experimental Methods is HEP
Oklahoma State University

September 11, 2017

Delta rays

- During ionization, the energy is transferred to electrons – what happens to these electrons?
  - if transferred energy is small, the electron is absorbed almost at the same point where the interaction occurred
  - if the interaction involved a lot of energy transfer (head-on or almost head-on collision), the electron may travel a significant distance before it loses its energy – these electrons are called delta rays (or delta ray electrons)
- If we model the passage of a particle though matter we want to distinguish between these situations
  - don't want to simulate creation of electrons of low energies – it's a waste of CPU time
  - it's important to simulate the passage of delta rays of high energies – they may escape the detector volume (so the deposited energy will be less than the one predicted by the Bethe formula), or they may produce additional ("secondary") ionization
- In reality, there is no clear boundary between continuous energy loss and delta ray production, but in models there is usually a parameter ("delta ray cut-off") above which the delta rays are considered real particles
  - the actual cut-off value depends on the physics problem, typical values are $10 - 100$ keV

Calculation of maximum electron energy

- Maximum energy transfer occurs in head-on collision

$$\begin{cases} p_0 = p_1 + p_e \\ E_0 + m_e = E_1 + E_e \end{cases}$$

- using $E_1 = \sqrt{M^2 + p_1^2}$ and squaring,
  $$E_0 + m_e = \sqrt{M^2 + (p_0 - p_e)^2} + E_e$$
  $$p_0 p_e = (E_0 + m_e)(E_e - m_e)$$
- using $E_e = \sqrt{m_e + p_e^2}$ and squaring,
  $$[(E_0 + m_e)^2 - p_0^2]E_e^2 - 2m_e(E_0 + m_e)^2 E_e + m_e^2[(E_0 + m_e)^2 + p_0^2] = 0$$
- this is a quadratic equation w.r.t. E_e, which has the solutions
  $$E_e = m_e \frac{(E_0 + m_e)^2 \pm p_0^2}{(E_0 + m_e)^2 - p_0^2}$$
- the '$-$' solution corresponds to the case without interaction
- the '$+$' solution is the one we are looking for

Energies of delta rays

- The maximum energy of a delta ray electron can be significant
  - For a proton ($M = 938.3$ MeV) $E_e = 5(29)$ MeV at incident particle momentum of 2(5) GeV
- Relation between the electron kinetic energy T_e and emission angle θ:

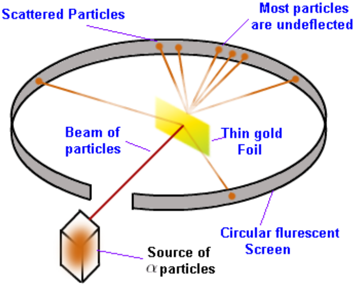
$$cos\theta = \frac{T_e/p_e}{T_e^{max}/p_e^{max}}$$

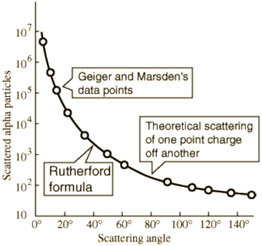
- In general, the probability of a head-on collision is very low, most delta rays are soft
  - for a $\beta = 1$ particle on average only one collision with $T_e > 10$ keV will occur along a path length of 90 cm of Ar gas

Particle scattering

- We discussed the quantitative picture of energy losses, but what about the path of the particle?
  - one would expect that due to interactions with matter, particle path would deviate from a straight line
- If the interaction probability is very small (so the probability to have more than one interaction is negligible), we have single scattering described by the Rutherford formula

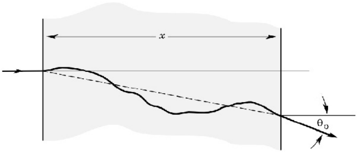
$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{z^2 e^4}{M^2 c^4 \beta^4} \frac{1}{\sin^4(\theta/2)}$$





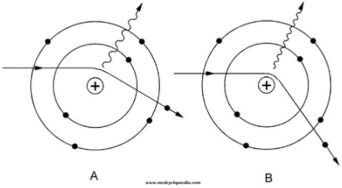
Multiple scattering

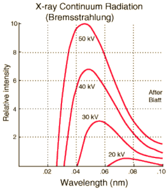
- If the number of interactions is large, it is multiple scattering – the process described by Molière theory (Phys. Rev. 89 (1256) 1953)
- for small deflection angles, multiple scattering is roughly Gaussian with a width θ_0 given by
$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p} z \sqrt{x/X_0}[1 + 0.038 \ln(x/X_0)]$$
- X_0 is a parameter called radiation length (more on that later)



Electron interactions

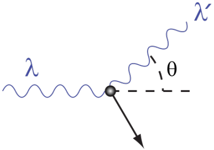
- At low energies, electrons lose energy due to ionization just like other charged particles
  - there are some differences between energy losses for electrons and positrons at low energies, e.g. annihillation is present for positrons but not for electrons
- At higher energies, the dominant mechanism of energy loss becomes bremsstrahlung (German: “braking radiation”)
  - in general, any accelerating charge radiates, and this leads to many phenomena (synchrotron radiation etc.)
  - Bremsstrahlung usually refers to particles decelerating in the medium



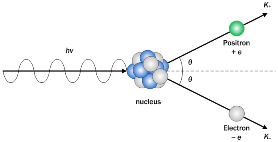


Photon interactions

- Photons don't have electric charge, so there is no ionization
  - instead of steady loss, photons undergo "catastrophic" interactions leading to significant energy loss in a single act
- At low energies, photons mostly lose energy due to Compton scattering



- As the energy increases, the photon energy loss becomes dominated by pair production
  - a photon can't just "decay" into a e^+e^- pair, it would violate the 4-momentum conservation – need presence of a nucleus



Radiation length

- Radiation length X_0: the mean distance over which a high energy electron loses all but $1/e$ of its energy by bremsstrahlung $-(dE/dx)_{\mathrm{brems}} = E/X_0$
- Besides being a characteristic length for bremsstrahlung, X_0 is a scaling parameter for multiple scattering (as we've seen) and also a characteristic length for the pair production by photons: $\sigma = \frac{7}{9}\frac{A}{X_0 N_A}$
- The radiation length defined as above is not exactly a constant since the energy loss (slightly) depends on the electron energy
- The definition implies that X_0 has units of length, but it is also common to multiply it by material density and measure X_0 in g/cm^2

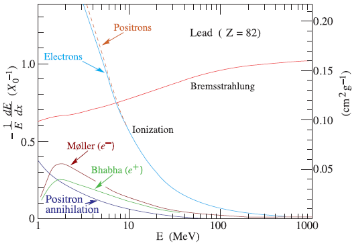
A practical approximation: a fit to data, <2.5% accuracy for all elements except He

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A}\left[Z(Z+1)\ln\frac{287}{\sqrt{Z}}\right]$$

for $A = 1$ g/mol, $4\alpha r_e^2 \dfrac{N_A}{A} = (716.4\ \mathrm{g/cm^2})^{-1}$

Critical energy

- The energy at which the energy loss of electrons due to ionization becomes equal to the one due to bremsstrahlung is called critical energy E_c



- When a high energy electron (photon) enters the absorber, it initiates an electromagnetic cascade as bremsstrahlung (pair production) generate lower energy photons (electrons)
- Eventually the energy of all involved electrons and photons falls below the critical energy and the rest of the energy is deposited through ionization (Compton scattering)