Passage of particles through matter

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Radiation length

- Radiation length X_0: the mean distance over which a high energy electron loses all but $1/e$ of its energy by bremsstrahlung $-(dE/dx)_{\rm brems} = E/X_0$
- Besides being a characteristic length for bremsstrahlung, X_0 is a scaling parameter for multiple scattering (as we've seen) and also a characteristic length for the pair production by photons: $\sigma = \frac{7}{9}\frac{A}{X_0 N_A}$
- The radiation length defined as above is not exactly a constant since the energy loss (slightly) depends on the electron energy
- The definition implies that X_0 has units of length, but it is also common to multiply it by material density and measure X_0 in g/cm^2

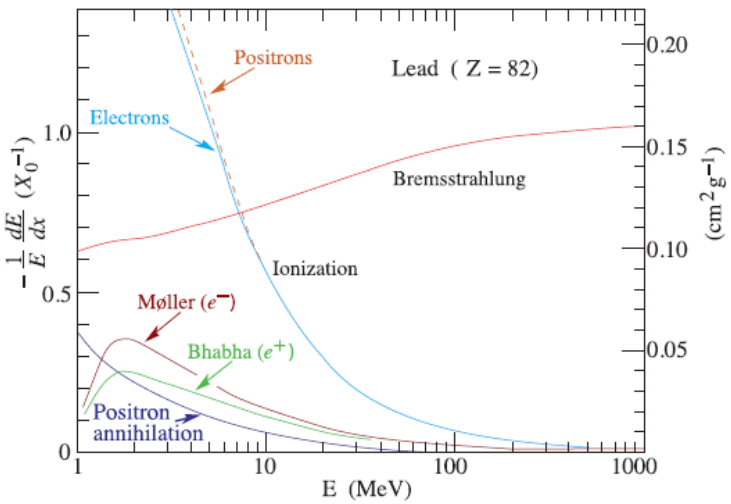
A practical approximation: a fit to data, <2.5% accuracy for all elements except He

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A}\left[Z(Z+1)\ln\frac{287}{\sqrt{Z}}\right]$$

for $A = 1$ g/mol, $4\alpha r_e^2 \frac{N_A}{A} = (716.4\ \mathrm{g/cm^2})^{-1}$

Critical energy

- The energy at which the energy loss of electrons due to ionization becomes equal to the one due to bremsstrahlung is called critical energy E_c



- When a high energy electron (photon) enters the absorber, it initiates an electromagnetic cascade as bremsstrahlung (pair production) generate lower energy photons (electrons)
- Eventually the energy of all involved electrons and photons falls below the critical energy and the rest of the energy is deposited through ionization (Compton scattering)

Nuclear interaction length

- As we've seen, high energy electrons and photons produce electromagnetic cascades, or "showers"
- Similarly, high energy hadrons (pions, kaons, protons, neutrons) produce "hadronic showers"
  - when hadrons hit nuclei, they create other particles due to strong interaction
- This process can be characterized by a typical length parameter called "nuclear interaction length" λ – the distance over which the initial number of hadrons reduces by a factor of $1/e$
  - for high Z materials, λ is much greater than X_0

material	X_0 [g/cm^2]	λ [g/cm^2]
H_2	63	52.4
Al	24	106
Fe	13.8	132
Pb	6.3	193

Nuclear interaction length – remarks

- Nuclear interaction length is particle dependent – e.g. λ for π^+ and π^- differ by $\sim$5%
- the total energy deposited in a hadronic shower is not equal to the incident particle energy – a lot of it is lost due to escaping neutrinos
- hadron showers allow to capture neutrons which don't have electric charge and don't lose energy by ionization

Range

- If we integrate the Bethe formula we'll get the mean distance travelled by a particle before it stops
  - this number is called range R

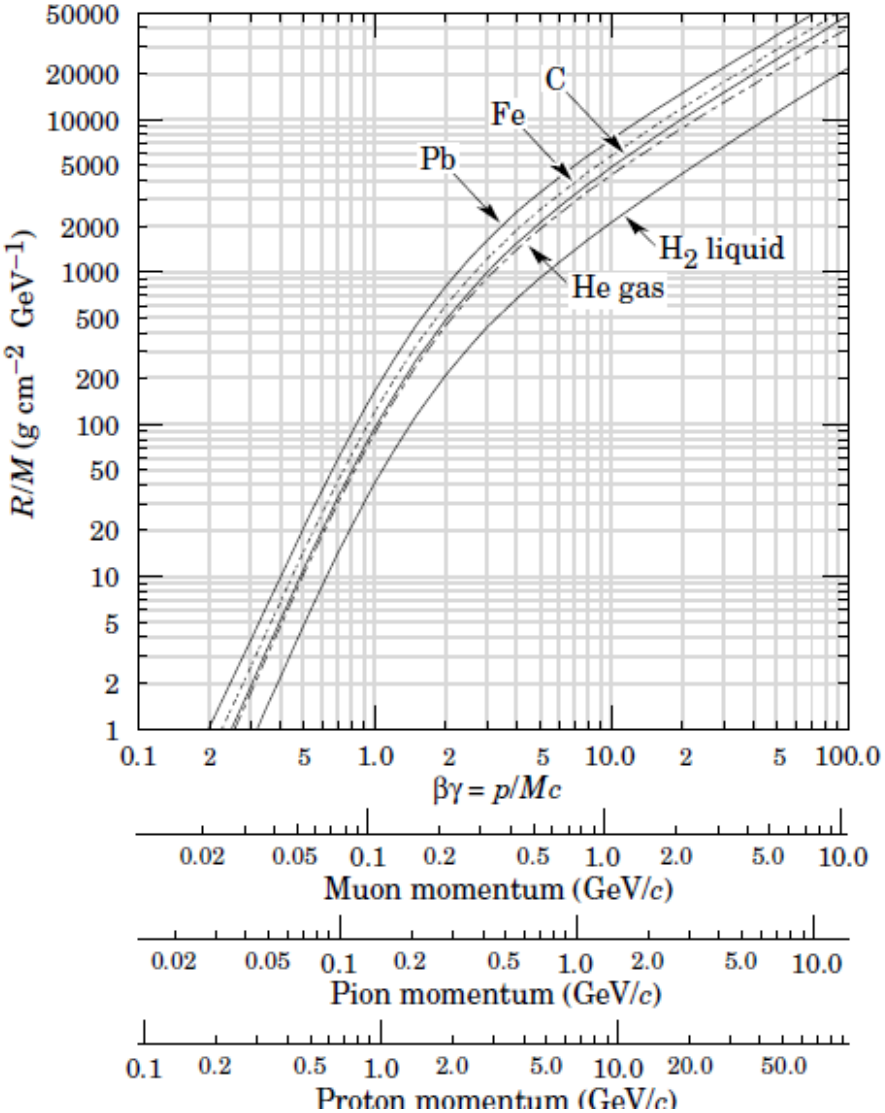
$$R = \int_0^R dx = -\int_{E_0}^0 \frac{dx}{dE/dx}$$

  - Range is a useful concept only for low energy hadrons and muons
- For non-relativistic particles, $-\frac{dE}{dx} = C\frac{Z}{A}\frac{z^2}{v^2}$ where C is a constant (here I assume that $\log v$ is a slowly changing function compared to $1/v^2$ and so it is included in the constant) and $E = m + T$, $T = \frac{1}{2}mv^2$

$$R = \frac{A}{CZmz^2}\int_0^{T_0} 2T\,dT = \frac{AT_0^2}{CZmz^2} = \frac{Amv_0^4}{4CZz^2}$$

- Range is usually given in terms of R/m (independent from particle mass m)

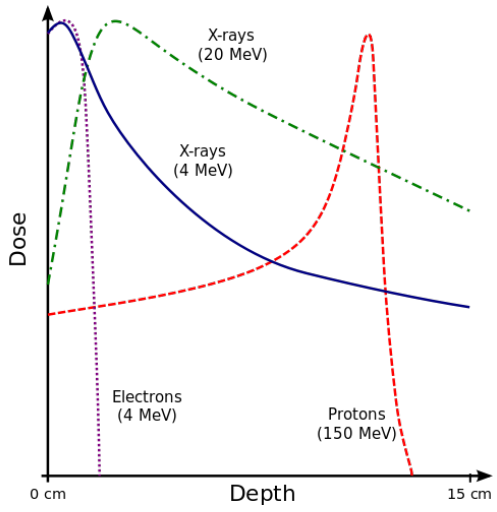
Range of heavy charged particles in various materials



Bragg curve

- As a heavy charged particle passes through matter, its energy decreases and the energy loss increases
  - most of the particle energy is deposited near the range R, leading to a sharp peak
- If energy losses were always equal to $|dE/dx|$ as described by the Bethe formula, the particles would always travel a distance R and then stop
  - in practice, the deposited energy fluctuates, so the peak gets smeared
  - Landau distribution is bound on the left side, so the deposition peak does not extend much to the right

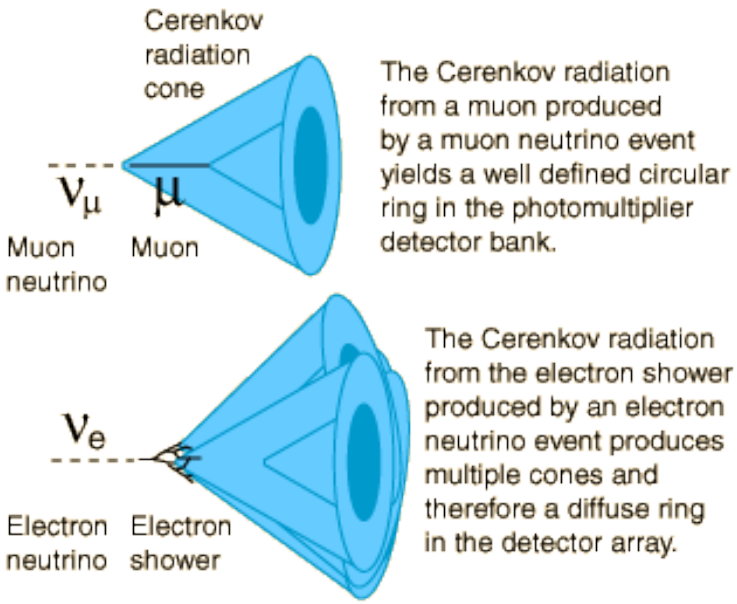
- The energy loss per unit length as a function of penetration depth is called the Bragg curve
  - knowing its exact shape is very important in particle therapy
- For electrons and photons, the energy losses are "catastrophic," they travel a certain distance and then get kicked out
  - the energy deposited by e/γ decreases as an exponent of the distance, there is no peak



Cherenkov radiation

- Cherenkov radiation is emitted when a charged particle goes with a speed larger than the speed of light in the medium
  - there is nothing that prevents a particle to do it as long as its speed is below c
- The effect is similar to a sonic boom created by a super sonic aircraft moving faster than the speed of sound
  - actually this is not exactly true: a sonic boom is a non-linear effect, Cherenkov radiation is linear

- The radiation is concentrated in a cone of a half angle $\theta = \arccos \frac{1}{n\beta}$, where n is the refractive index
  - by measuring the size of the cone, it's possible to calculate the speed of the particle
- Particles in a electromagnetic cascade can produce what is called "coherent Cherenkov radiation"



Transition radiation

- When a particle with charge ze goes from vacuum to a medium with plasma frequency ω_p, it radiates energy $\Delta E = \alpha z^2 \gamma \hbar \omega_p / 3$

  $\hbar\omega_p = \sqrt{\rho[\mathrm{g/cm^3}](Z/A)} \times 28.81\ \mathrm{eV}$
  - Cherenkov radiation occurs when a particle goes in a medium with constant n; transition radiation requires a change of n
- Transition radiation can be used to determine particle's γ
  - A popular use is to discriminate between electrons and heavy particles
- The amount of energy deposited in a single transition is very low, so for practical purposes one has to set up a stack of radiators separated by gas gaps