Chapter 17

Electric Potential
Force and Work: Reminder

Reminder from Mechanics:

- if there is a force acting on an object (e.g. electric force), this force may do some work when the object moves.
\[ W_{a \rightarrow b} = Fd \cos \varphi \]

that’s why direction matters

a: initial point

b: final point

Work can be positive (\( \varphi < 90^\circ \)) or negative (\( \varphi > 90^\circ \))
Conservative Force

• If a force is conservative, there is something called potential energy $U$ which changes from point to point.

$$W_{a\rightarrow b} = U_a - U_b$$

b: final point

Work does not depend on the path!

Electric Potential
Work-Energy Theorem

• Change in kinetic energy = work

\[ K_b - K_a = W_{a \to b} \]

\[ K_b - K_a = U_a - U_b \]

\[ K_a + U_a = K_b + U_b \]

total energy doesn’t change
Electric Potential Energy

• If charge $q$ is placed in electric field $E$, the force acting on it is $F = qE$

$$U_a - U_b = W_{a\rightarrow b} = qEd \cos \varphi$$

\[ +q \rightarrow E \rightarrow v_1 \rightarrow v_2 \]
Electric Potential

• Force $\rightarrow$ Field = Force divided by probe charge $q$
  – Field is independent of $q$

• Potential energy $\rightarrow$ Potential = Potential energy divided by probe charge $q$
  – Potential is independent of $q$

\[ V = \frac{U}{q} \]

$[V] = \text{Volt}$ \hspace{1cm} 1 \text{ V} = 1 \text{ J} / \text{C}$
Electric Potential Difference

• Like potential energy, potential itself is meaningless, what makes sense is the potential difference

$$V_{ba} = -\frac{W_{a\rightarrow b}}{q}$$

potential difference ("voltage") between points a and b

$$V_{ba} = V_b - V_a$$

charge accelerates

a: high potential
b: low potential
**Relation between Electric Potential Difference and Electric Field**

\[ W_{a \rightarrow b} = q Ed \cos \varphi \]

\[ V_{ba} = - \frac{W_{a \rightarrow b}}{q} \]

\[ V_{ba} = -Ed \]

Consider a probe charge moving parallel to electric field lines. 

\[ d \quad \vec{E} \]

a: high potential

b: low potential
Field of Point Charge

- Charge produces electric field which can act on another charge.

There is potential energy due to interaction of two charges.

- Field is non-uniform.
- Field is not constant.

\[ E = k \frac{Q}{r^2} \]
Work in Field of Point Charge

\[ E = k \frac{Q}{r^2} \]

\[ W = qEd \cos \phi \]

\[ \phi = 0, \text{ OK} \]

\[ E \text{ varies with } x, \text{ not OK!} \]

Need calculus to compute result
Work in Field of Point Charge

\[ W_{a\rightarrow b} = kQq \left( \frac{1}{a} - \frac{1}{b} \right) = k \frac{Qq}{a} - k \frac{Qq}{b} \]

\[ U = k \frac{Qq}{r} \]

Formula valid for any combination of q, q’ signs
More about potential energy

- $U$ is always defined w.r.t. $U$ at some point chosen by convention (only $U_a - U_b$ has physical meaning)
  - our choice: $U=0$ at $r=\infty$
- What if there are more than one charge?

$$U = kq \left( \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots \right)$$

![Electric Potential](image)
Potential of a Point Charge

\[ V = \frac{U}{q} = k \frac{Q}{r} \]

For many point charges:

\[ V = k \left( \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots \right) \]

• Both potential and electric field are independent from test charge
• Potential is a scalar, field is a vector
Equipotential Surfaces

electric field line: imaginary line which in each point is tangent to electric field vector

equipotential surface: potential is the same at every point

• two equipotential surfaces never touch or intersect
• equipotential surfaces are mutually perpendicular to field lines
Uniform Field

\[ V = \frac{U}{q} = E_y \]

where

- \( V \) is the electric potential,
- \( U \) is the electric potential energy,
- \( q \) is the charge,
- \( E_y \) is the component of the electric field in the y-direction.

From this, it follows that:

- \( V = \text{const} \) \( \Rightarrow \) \( y = \text{const} \)

Equipotential surfaces are planes perpendicular to the y-axis.
Point Charge

\[ V = k \frac{Q}{r} \]

\[ V = \text{const} \Rightarrow \]

\[ r = \text{const} \]

equipotential surfaces = spheres with centers at origin
Dipole

If charges +q and −q are placed at y=+a and y=−a then plane y=0 is an equipotential surface

Proof: \[ V = k \left( \frac{Q}{r_1} - \frac{Q}{r_2} \right) \]

at y=0 \( r_1 = r_2 \), so \( V = 0 \)
Potential Gradient

• The magnitude of the electric field at any point on an equipotential surface = rate of change of potential

\[ E = -\frac{\Delta V}{\Delta s} \]

as a point moves along electric field, potential decreases

electric field is a vector, so what about direction?

\[ \Delta s = (\Delta x, \Delta y, \Delta z) \]

\[ \vec{E} = \left( -\frac{\Delta V}{\Delta x}, -\frac{\Delta V}{\Delta y}, -\frac{\Delta V}{\Delta z} \right) \]
Capacitor and capacitance

- Capacitor = device which stores electric charge
  - Capacitor ≠ battery!

- Capacitor = two conductors separated by an insulator
  - generally, conductor 1 has charge $q_1$, and conductor 2 has charge $q_2$, but we’ll always assume $q_1=Q=-q_2$, so the net charge is 0

- Capacitance $C$ = capacitor’s ability to store charge

\[
\begin{align*}
C &= \frac{Q}{V} \\
[C] &= \text{Farad}, \ 1 \text{ F} = 1 \text{ C / V}
\end{align*}
\]
Parallel-Plate Capacitor

\[ \sigma = \frac{Q}{A} \]  
- surface charge density

\[ E = \frac{\sigma}{\varepsilon_0} \]  
- this can be proved using Gauss’ law

\[ V = Ed \]

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ \varepsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \]
Dielectrics

• If there is some material between the capacitor plates, the capacitance increases due to polarization

\[ K = \frac{C}{C_0} \quad V = \frac{V_0}{K} \quad E = \frac{E_0}{K} \]

K = dielectric constant (a pure number depending on material)
If we insert material inside capacitor charged with charge Q, the voltage decreases by K
Electric Field Energy

- Potential energy of a capacitor = work needed to charge it
  - If there is already charge $Q$ and we want to add more charge $\Delta Q$ then we need to do work $\Delta W = V \Delta Q$

$$\Delta W = V \Delta Q = \frac{Q \Delta Q}{C}$$

$$U = W_{total} = \frac{V}{2} Q = \frac{Q^2}{2C} = \frac{CV^2}{2}$$
Electric Field Energy Density

- Energy density \( u = \text{energy} \ U \text{ per unit volume} \ v \)

\[
v = Ad
\]

\[
u = \frac{U}{v} = \frac{CV^2}{2Ad}
\]

\[
C = \frac{\varepsilon_0 A}{d}
\]

\[
V = Ed
\]

\[
u = \frac{\varepsilon_0 E^2}{2}
\]

there is nothing related to a capacitor in this formula – it’s also valid for a field in vacuum!